



Find the magnitude of field components on the  $z$ -axis as a function of  $L$

$E_r = 0$  due to symmetry

$$E_z = \int_{-h/2}^{+h/2} \int_0^R \frac{\sigma}{2 \cdot \epsilon \cdot h \cdot (r^2 + (L-z)^2)} \cdot r \cdot \cos \phi \cdot dr \cdot dz = \int_{-h/2}^{+h/2} \int_0^R \frac{\sigma \cdot (L-z)}{2 \cdot \epsilon \cdot h \cdot (r^2 + (L-z)^2)^{3/2}} \cdot r \cdot dr \cdot dz$$

$$E_z = \frac{\sigma}{2 \cdot \epsilon \cdot h} \cdot \int_{-h/2}^{+h/2} \left( \frac{L-z}{\sqrt{(L-z)^2}} - \frac{L-z}{\sqrt{R^2 + (L-z)^2}} \right) \cdot dz$$

$$E_z = \frac{\sigma}{2 \cdot \epsilon \cdot h} \cdot \left( \sqrt{R^2 + L^2 - h \cdot L + \left(\frac{h}{2}\right)^2} - \sqrt{L^2 - h \cdot L + \left(\frac{h}{2}\right)^2} - \sqrt{R^2 + L^2 + h \cdot L + \left(\frac{h}{2}\right)^2} + \sqrt{L^2 + h \cdot L + \left(\frac{h}{2}\right)^2} \right)$$

Note that if  $L$  is set to zero we get the field at the center of the disk  $E_z = 0$

If one takes the limit as  $h$  goes to zero, we approach the thin disk with surface charge density.

$$E_z = \frac{\sigma}{2 \cdot \epsilon} \cdot \left( 1 - \frac{L}{\sqrt{R^2 + L^2}} \right) \text{ Once the limit is taken, the answer at } L=0 \text{ is indeterminate.}$$