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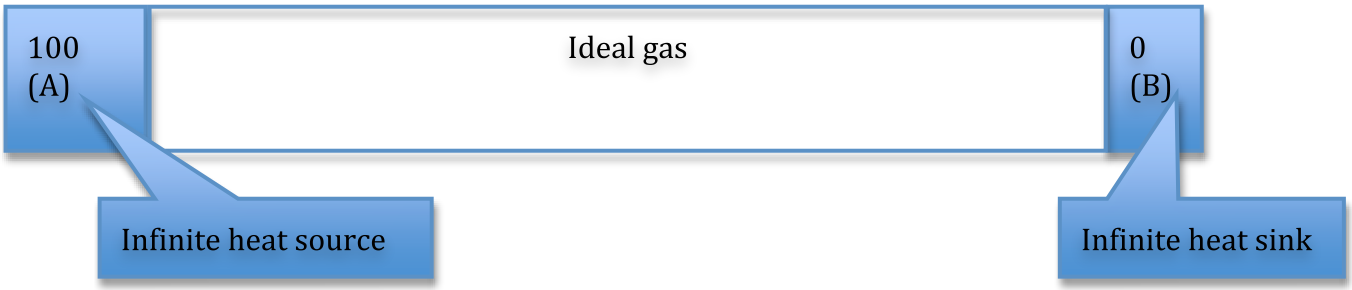
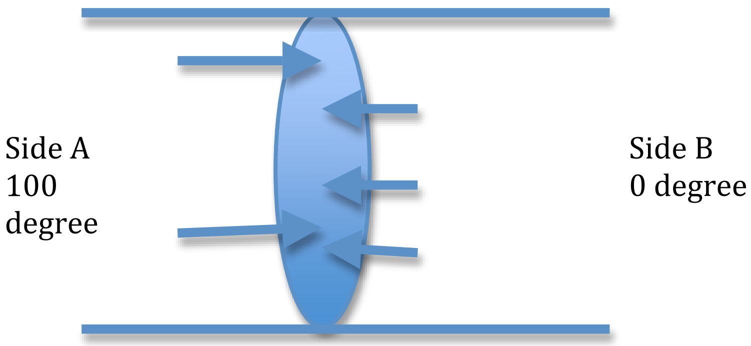
Temperature and Pressure gradient in Ideal Gas



Consider a circular tube with one side is maintained at 100 degree Celsius and other side is maintained at 0 degree Celsius. Temperature at both ends is maintained at constant temperature using an infinite heat source at A and infinite heat sink at B. Faces facing A and B (flat faces) is perfect conductor. Circular face is perfect insulator. Tube is filled with ideal gas.
What will be the temperature and pressure gradients inside the tube?

1. Will heat flow from A to B?
2. Will the Temperature at A higher than temperature that at B?
3. Will the Pressure at A higher than pressure that at B?
4. Will the Density at A higher than that at B?



Consider the cross section at an arbitrary point in the tube. Consider the kinetic theory of gas. Assume the all particles of the gas are of same weight.

- Temperature in the tube touching side A shall be 100 and temperature in the tube touching side B shall be 0. Hence there will be a temperature gradient.
- Hence heat will flow from A to B.
- Number of particles passing from A to B will be equal to number of particles passing from B to A.
- Number of particles passing through the cross section is proportional to density of the particles (d) and velocity of the particles ($\bar{u}$). That means throughout the tube, the $d\bar{u}$ will be constant.
- Temperature of ideal gas is proportional to the square of the velocity of the particles. Hence velocity of gas particles at side A shall be higher than velocity at side B.

- As dū is constant, then density at side A shall be less than density at side B.
- According to kinetic theory, Pressure of a ideal gas is due to the particles of the gas hitting the boundary of the container. Pressure will be equal to the rate of change of momentum at the boundary. Pressure is directly proportional to the number of particles hitting the boundary and momentum of individual particles. Number of particles hitting the boundary is proportional to dū and momentum is proportional to velocity of the particles. Hence pressure is proportional to dū and ū.
- Since dū is constant across the tube, pressure is directly proportional to velocity of the particles (ū). As we have seen velocity is directly proportional to the square root of temperature.
- Hence pressure inside tube shall be higher at side A and lower at side B.

This is in fact analogous to Bernoulli's principle (which states that there will be a pressure gradient along with a velocity gradient).
From this we can conclude that:

1. **There will be pressure gradient along with a temperature gradient. The pressure gradient is directly proportional to the gradient of square root of temperature.**
2. **There will be density gradient along with temperature gradient. The density gradient is inversely proportional to the square root of the temperature.**