To support this formula we use Merten's 3rd theorem:

$\lim\limits_{n\to\infty} \prod\limits_{2\le p\le n} \frac{p-1}{p} \cdot \log(n) = e^{-\gamma}$, with the Euler-Mascheroni constant $\gamma = 0.5772156649\ldots$,

from which we deduce the asymptotic equivalences

$$\frac{1}{2}\cdot e^{\gamma}\cdot \prod_{2\le p<\sqrt{n}} \frac{p-1}{p} \sim \frac{1}{\log(n)}, \quad \text{and, therefore} \quad \frac{\pi(n)}{n} \sim \frac{1}{2}\cdot e^{\gamma}\cdot \prod_{2\le p<\sqrt{n}} \frac{p-1}{p} \qquad (2.1)$$

There is something worth noting about these formulas, in which the factor

$$\prod_{2\le p<\sqrt{n}} \frac{p-1}{p} \qquad (2.1.1)$$

expresses the density, at the limit $n \to \infty$, of the integers which are not divisible by any of the prime numbers $2 \le p < \sqrt{n}$. It is remarkable, that, at the limit, the density of the primes among these integers can be obtained by multiplying this factor by a constant, $e^{\gamma}/2 = 0.890536209\ldots$.

Numbers q ($\sqrt{n} < q < n$) which lend themselves to a Goldbach Partition must satisfy the two conditions:

$q \not\equiv 0 (\operatorname{mod} p)$ and $q \not\equiv n (\operatorname{mod} p)$ for all prime numbers p ($2 \le p < \sqrt{n}$) (2.2.1)

To estimate their density, we have to modify the formula (2.1) to :

$$\frac{G(n)}{n} \sim \frac{1}{2}\cdot e^{\gamma}\cdot \prod_{\substack{2\le p<\sqrt{n} \\ n\equiv 0(\operatorname{mod} p)}} \frac{p-1}{p} \cdot \prod_{\substack{3\le p<\sqrt{n} \\ n\not\equiv 0(\operatorname{mod} p)}} \frac{p-2}{p} \qquad (2.2.2)$$

Let's note, in analogy to (2.1.1), that the expression

$$\prod_{\substack{2\le p<\sqrt{n} \\ n\equiv 0(\operatorname{mod} p)}} \frac{p-1}{p} \cdot \prod_{\substack{3\le p<\sqrt{n} \\ n\not\equiv 0(\operatorname{mod} p)}} \frac{p-2}{p} \qquad (2.2.2.1)$$

represents the density, at the limit $n \to \infty$, of the integers q fulfilling the conditions (2.2.1). It seems reasonable to assume, that, at the limit, the density of the primes among these integers can be obtained by multiplying (2.2.2.1) with the same constant, $e^{\gamma}/2$.

Now, (2.2.2) can be written in the form :

$$G(n) \sim \pi(n)\cdot \prod_{\substack{3\le p<\sqrt{n} \\ n\equiv 0(\operatorname{mod} p)}} p-1 \cdot \prod_{\substack{3\le p<\sqrt{n} \\ n\not\equiv 0(\operatorname{mod} p)}} p-2 \Bigg/ \prod_{3\le p<\sqrt{n}} p-1 \qquad (2.2.3)$$

Simplifying the quotient, and counting partitions into prime numbers q and $n-q$, with $q \le n/2$, we obtain the formula (2a).

Obviously, the formulas (2a) and

$$g_{alt}^{'}(n) = \frac{1}{2} \cdot \frac{n}{\log(n)} \cdot \prod_{\substack{3 \le p < \sqrt{n} \\ n \not\equiv 0 (\bmod p)}} \frac{p-2}{p-1} \quad \text{, and} \qquad (2b)$$

$$g_{alt}^{''} = \frac{n}{4} \cdot e^{\gamma} \cdot \prod_{\substack{2 \le p < \sqrt{n} \\ n \equiv 0 (\bmod p)}} \frac{p-1}{p} \cdot \prod_{\substack{3 \le p < \sqrt{n} \\ n \not\equiv 0 (\bmod p)}} \frac{p-2}{p}$$

(2c)

are equivalent, as are the formulas (1.1, 1.2), and (2) :

$$\frac{g_{alt}}{g_{HL}} = \left[\frac{\pi(n)}{n/\log(n)} \right] \cdot \left[\frac{1}{1/\log(n)} \cdot \frac{e^{\gamma}}{2} \cdot \prod_{2 \le p < \sqrt{n}} \frac{(p-1)}{p} \right] \cdot \left[\prod_{\sqrt{n} < p < \infty} \frac{(p-1)^2}{p \cdot (p-2)} \right] \sim 1,$$

each of the three factors in square brackets tending towards unity.