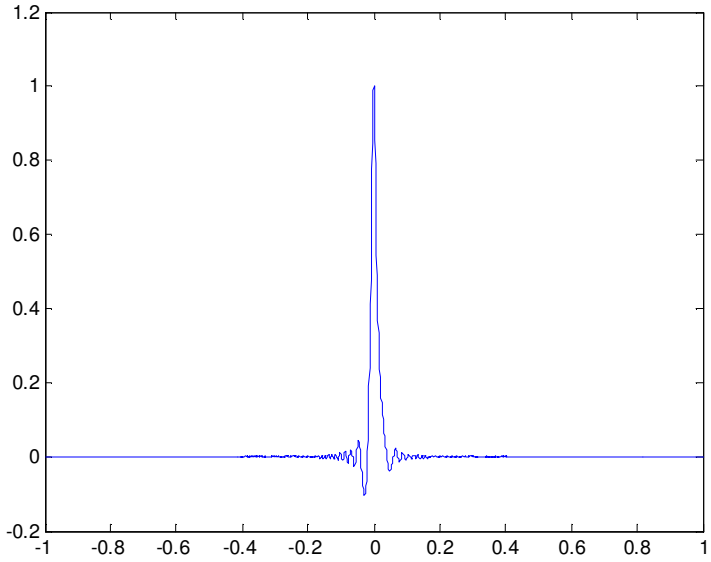
$$\int_{-\infty}^{\infty} \underbrace{\frac{1}{1+ax^2}\exp\left[\frac{jk}{2L}(x^2-2xr)\right]}_{f(x)} dx$$

a is vaiable $a \sim 10^4$,
L is variable, $L \sim 10^3$,
k = 2pi/lamda,
lamda = $1.55\text{x}10^{-6}$,
r is variable $r \sim 0.01$, $j = \sqrt{-1}$

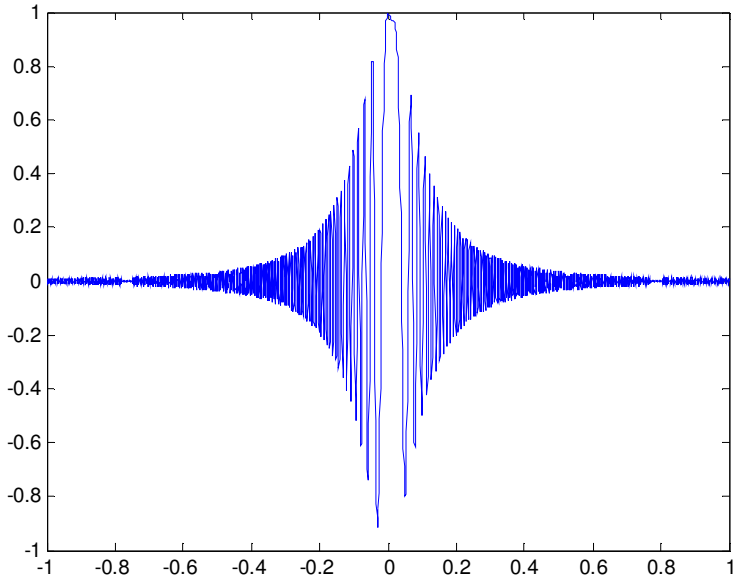
SPM (Stationary Phase Method)

f (x)



For a = 10^4 oscillation disappear
Numerical solution = 0.0205 + 0.0046i
Approximately SPM solution = 0.0164 + 0.0108i

$f(x)$



For a = 10^2 oscillation appear

Numerical solution = 0.0330 + 0.0207i

Approximately SPM solution = 0.0325 + 0.0215i

This my Matlab code for numerical solution

```
%clc; clear all
lamda=1.55e-6;
k=2*pi/lamda;
a=1e4;
r=1e-2;L=1000;
f=@(x)(1./(1+a*x.^2).*exp(i*k/(2*L)*(x.^2-2*x*r)));%% lorentz beam
integral by numerical solution
numerical=quade(f,-10,10)% quade is special numerical integrator
analytical=sqrt(2*pi/(-i*k/L))*(1/(1+a*r^2))*exp(-i*(k/(2*L)*r^2))%
lorentz beam integral by SPM solution
x=-1:0.001:1;
function1=(1./(1+a*x.^2).*exp(i*k/(2*L)*(x.^2-2*x*r)));
plot(x,function1)
```