Question 1:
$$0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$$
is an exact short sequence,in order to prove

$$\cdots \to H^q(A) \xrightarrow{f^*} H^q(B) \xrightarrow{g^*} H^q(C) \xrightarrow{d^*} H^{q+1}(A) \to \cdots$$

is an exact long cohomology sequence,we need to prove

$$\mathrm{Im} f^* = \ker g^*$$

,I can prove
$$\mathrm{Im} f^* \subset \ker g^*$$

because
$$\mathrm{Im} f = \ker g$$

,but how to prove $\mathrm{Im} f^* \supset \ker g^*$?

Question 2: What makes the cohomology groups different by digging 2 points of R^2 from that of R^2 ?How does the closed and exact differential forms change?
    Thank you!