A question in random variables and random processes

Let

$$X(t) = A\cos(\omega_0 t + \Phi)$$

Be a random process, in which A and ω_0 are positive constants. Let Φ be a r.v. with pdf

$$f_\Phi(\varphi) = \begin{cases} \dfrac{1}{2\pi}, & |\varphi| < \pi \\ 0, & \text{otherwise} \end{cases}$$

and let

$$Y(t) = X^2(t)$$

be another r.p., which performs a non-linear transform of $X(t)$.
Is the expectancy of $Y(t)$ constant (i.e. does not depend on t)?

Formal Solution

Problem 7

$$X(t) = A\cos(\omega_0 t + \Phi) \qquad f_\Phi(\varphi) = \begin{cases} \frac{1}{2\pi} & |\varphi| < \pi \\ 0 & |\varphi| > \pi \end{cases}$$

$$E(Y(t)) = E(X^2(t)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} A^2\cos^2(\omega_0 t + \varphi)\,d\varphi =$$

$$= \frac{A^2}{2\pi}\int_{-\pi}^{\pi}\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t + 2\varphi)\right)d\varphi = \frac{A^2}{2};$$

I just cannot understand why

$$\mathbb{E}\left[X^2(t)\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} A^2\cos^2(\omega_0 t + \varphi)\,d\varphi \ .$$

is applicable here, because what is vitually meant by this equation is that

$$\mathbb{E}\left[X^2(\varphi)\right] = \int_{-\infty}^{\infty} X^2(\varphi)\cdot f_\Phi(\varphi)\,d\varphi \ ,$$

but the pdf in the integrand is the pdf of Φ and not of X. In addition, X is function of t, but it depends also on φ. Can I use the definition of the expectancy for both independent variables whatsoever?

Thanks! ☺