On the Nonexistence of Quasiperfect Numbers

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A quasiperfect number is a number which the sum of its divisors is equal to one more than twice the number. All quasiperfect numbers (if they exist) would be of the form

$$\sigma(n) = 2n + 1, \tag{1}$$

where *n* is some integer. However, It is known (Cattaneo [1]) that if there are any quasiperfect numbers, they must be odd perfect squares. Hence if *n* is a quasiperfect number, then it must also be of the form

$$n = 4x^2 - 4x + 1, \tag{2}$$

where $x \in Z^+$. With this understanding of *n*, we must now consider the index function or

$$h(n) = \frac{\sigma(n)}{n} \tag{3}$$

If $h(n) < 2$, then *n* is deficient.

If $h(n) = 2$, then *n* is perfect.

If $h(n) > 2$, then *n* is abundant.

If *n* is a quasiperfect number, then *h(n)* will be greater than 2, and its index function will be of the form

$$h(n) = \frac{2n+1}{n} \tag{4}$$

Which is equivalent to the following equations,

$$h(n) = 2 + \frac{1}{n} \tag{5}$$

$$h(n) - 2 = \frac{1}{n} \tag{6}$$

Because *h(n)* is abundant, *h(n)-2* is always positive. Now, because *n* must also fit the form of

$$n = 4x^2 - 4x + 1, \tag{7}$$

where $x \in Z^+$.

The index function of can now be written in the form

$$4x^2 - 4x + 1 = \frac{1}{h(n)-2} \tag{8}$$

We now subtract one from both sides, which creates the formula

$$4x^2 - 4x = -\frac{h(n)+1}{h(n)-2}, \tag{9}$$

However $4x^2 - 4x$ is always positive and $-\frac{h(n)+1}{h(n)-2}$ is always negative, therefore

$$4x^2 - 4x \neq -\frac{h(n)+1}{h(n)-2}, \tag{10}$$

But because (9) is a necessary condition for the condition for a quasiperfect number, a quasiperfect number cannot exist.

References

1. P. Cattaneo. "*Sui numeri quasiperfetti.*" Boll. Un. Mat. Ital. (3), 6 (1951):

59-62.