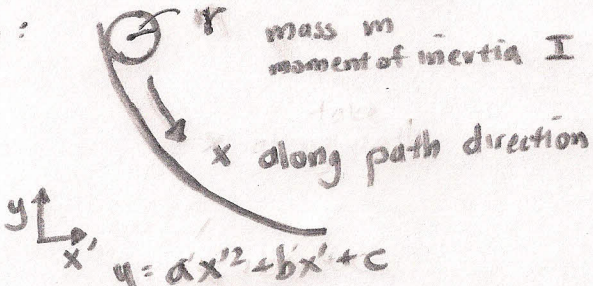
Problem Statement

I want to be able to plot a ball/cylinder's trajectory on a curved path given that it rolls w/o slip

Given:



$y(x)$: height of ball as function of path length travelled

Approach: Lagrangian Mechanics + Constraint

Roll w/o slip constraint: $f(x,\phi) = x - r\phi = 0$

$\mathcal{L} = T + V = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\phi}^2 + mgy(x)$

$y = ax + b'$

$\frac{\partial f}{\partial x} = 1, \quad \frac{\partial f}{\partial \phi} = -r$

E.L.: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} - \lambda\frac{df}{dq_i} = 0$

x: $\frac{\partial L}{\partial \dot{x}} = m\dot{x}, \quad \frac{\partial L}{\partial x} = mga'$

$m\ddot{x} - mga' - \lambda = 0$

ϕ: $\frac{\partial L}{\partial \dot{\phi}} = I\dot{\phi}, \quad \frac{\partial L}{\partial \phi} = 0$

$I\ddot{\phi} + \lambda r = 0$

$m\ddot{x} - mga' - \lambda = 0$

$I\ddot{\phi} + \lambda r = 0 \rightarrow \lambda = -\frac{I\ddot{\phi}}{r}$

$x - r\phi = 0 \rightarrow x = r\phi \rightarrow \ddot{x} = r\ddot{\phi} \rightarrow \ddot{\phi} = \frac{\ddot{x}}{r}$

$m\ddot{x} - mga' + \frac{I\ddot{x}}{r^2} = 0$

$\ddot{x} = \frac{mga'}{\left(m + \frac{I}{r^2}\right)}$