MA 315, Spring 2012

Homework VIII

Due Wednesday, April 4, 2012 by midnight

1. Let $f : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be the function defined by $f(t) = (3t, 3t + 1)$. Let B denote the subset of $\mathbb{Z} \times \mathbb{Z}$ defined by $B = \{(5m, 5m + 1) : m \in \mathbb{Z}\}$. Determine $f^{-1}(B)$. This means that you should define a set S with the property that $S = f^{-1}(B)$. In addition, your definition of S should make no mention of the function f.

2. Let $f : X \to Y$. For each element $b \in Y$, let $Y_b = f^{-1}(\{b\})$. Prove that the Y_b's have the following property:

   (a) If b and c are distinct elements of Y, then $Y_b \cap Y_c = \emptyset$.

   (b) $X = \bigcup_{b \in Y} Y_b$.