Distribution of Points

Mech 1.

(a) 3 points

Using the expression for the energy stored in a spring:

$U = \frac{1}{2} kx^2$ — 1 point

Substituting:

$U = \frac{1}{2}\left(400\ \frac{\text{N}}{\text{m}}\right)(0.5\ \text{m})^2$ — 1 point

$U = 50$ J — 1 point

(b) 5 points

For recognition of conservation of energy or work-energy theorem — 1 point

Kinetic energy of block C: $K = \frac{1}{2} m_C v_C^2$ — 1 point

Work done (or energy dissipated by) friction: $W_f = \mu F_n d$ — 1 point

$K = U - W_f$

$\frac{1}{2} m_C v_C^2 = U - \mu m_C g d$ — 1 point

Solving for v_C:

$$v_C = \sqrt{\frac{2}{m_C}(U - \mu m_C g d)}$$

Substituting:

$$v_C = \sqrt{\frac{2}{4\ \text{kg}}(50\ \text{J} - (0.4)(4\ \text{kg})(10\ \text{m/s}^2)(0.5\ \text{m}))}$$

$v_C = 4.58$ m/s — 1 point

(Full credit also awarded for correct alternate solution computing $\int F_{net}\, dx$, where $F_{net} = kx - \mu F_n$, to find the kinetic energy, and then computing the speed.)

Mech 1. (continued)

(c) 3 points

For any statement of conservation of momentum — 1 point

$m_C v_C = (m_C + m_D) v_f$ — 1 point

Solving for v_f:

$v_f = m_C v_C/(m_C + m_D)$

Substituting:

$v_f = (4\ kg)(4.58\ m/s)/(4\ kg + 2\ kg)$ — 1 point

$v_f = 3.05\ m/s$

(d) 3 points

The blocks come to rest when all their kinetic energy has been dissipated, i.e. ΔKE = Work done by frictional force — 1 point

$\frac{1}{2}(m_C + m_D) v_f^2 = \mu (m_C + m_D) g d$ — 1 point

Solving for d:

$d = v_f^2/2\mu g$

Substituting:

$d = (3.05\ m/s)^2/(2)(0.4)(10\ m/s^2)$

$d = 1.16\ m$ — 1 point

(Alternate Solution) — (Alternate Points)

$\sum F = ma$ — (1 point)

$$a = \frac{\sum F}{m} = \frac{\mu(m_C + m_D)g}{(m_C + m_D)} = \mu g = (0.4)(10\ m/s^2) = 4\ m/s^2$$

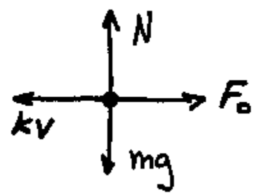
$v^2 = v_o^2 + 2ad$ (or other appropriate kinematic equations) — (1 point)

$$d = \frac{v^2 - v_0^2}{2a} = \frac{0 - (3.05\ m/s)^2}{2(-4\ m/s^2)} = 1.16\ m$$ — (1 point)

UNITS: For correct units on all answers — 1 point

Mech 2.

(a) 3 points



1 point for F_0 correctly drawn and labeled | 1 point
1 point for kv correctly drawn and labeled | 1 point
1 point for N and mg correctly drawn and labeled | 1 point

(b) 3 points

$F_{net} = ma$ — 1 point

But $F_{net} = F_0 - kv$, therefore: — 1 point

$F_0 - kv = ma$

Solving for a:

$a = (F_0 - kv)/m$ — 1 point

(c) 5 points

$a = \frac{dv}{dt}$ — 1 point

Using the equation from part b:

(1) $\frac{dv}{dt} = \frac{(F_0 - kv)}{m}$

Re-arranging and integrating:

(2) $\int \frac{dv}{F_0 - kv} = \int \frac{1}{m}\,dt$ — 1 point

Changing variables by letting $u = F_0 - kv$, $du = -k\,dv$:

$-\frac{1}{k} \int \frac{du}{u} = \int \frac{1}{m}\,dt$

$\ln(F_0 - kv) - \ln C = -\frac{k}{m}t$, where C is a constant

$v = \frac{1}{k}\left(F_0 - Ce^{-kt/m}\right)$

(these steps together: 1 point)

Distribution of Points

Mech 2. (continued)
(c) (continued)

To evaluate C, use initial conditions $t = 0$, $v = 0$ — 1 point

$C = F_0$

so $v = \frac{F_0}{k}\left(1 - e^{-kt/m}\right)$ — 1 point

Equation (2) can also be integated using limits 0 and v for the left-hand side and 0 and t for the right-hand side to obtain the same answer for full credit.

(Alternate Method to solve equation (1)) — (Alternate points)

Recognizing that the solution will be in exponential form, try:

$v = Ae^{Bt} + C$, where A, B, and C are constants — (1 point)

Substituting into equation (1)

$ABe^{Bt} = \frac{F_0}{m} - \frac{k}{m}(Ae^{Bt} + C)$

$ABe^{Bt} = \left(\frac{F_0}{m} - \frac{kC}{m}\right) - \frac{kA}{m}e^{Bt}$ — (1 point)

Equating coefficients to evaluate B and C,

$B = -\frac{k}{m}$, $C = \frac{F_0}{k}$

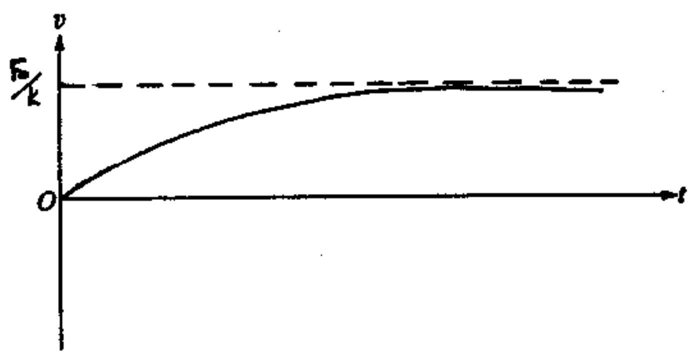
Therefore, $v = Ae^{-kt/m} + \frac{F_0}{k}$

To evaluate A, use initial conditions $t = 0$, $v = 0$ — (1 point)

$A = -\frac{F_0}{k}$

so $v = \frac{F_0}{k}\left(1 - e^{-kt/m}\right)$ — (1 point)

(d) 2 points



For correct maximum value F_0/k — 1 point

For correct shape of curve — 1 point

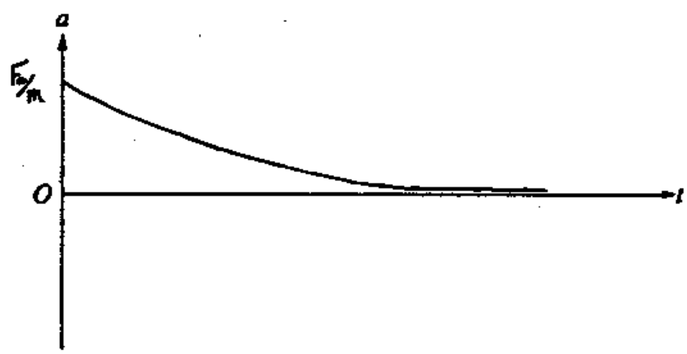
Mech 2. (continued)

(e) 2 points

Differentiating the expression for velocity from part (c) to get the expression for acceleration:

$$a = \frac{dv}{dt} = \frac{F_0}{k}\left[0 - (-k/m)\ e^{-kt/m}\right]$$

$$a = \frac{F_0}{m}\ e^{-kt/m}$$



For correct initial value F_0/m — 1 point

For correct shape of curve — 1 point

(Alternate Solution)

From part (b):

$$a = \frac{1}{m}\ (F_0 - kv)$$

Substituting for v from part (c):

$$a = \frac{F_0}{m} - \frac{k}{m}\frac{F_0}{k}\ (1 - e^{-kt/m})$$

$$a = \frac{F_0}{m}\ e^{-kt/m}$$

Mech 3.

(a) 4 points

$\tau = rF$ — 1 point

$\sum\tau = 0$ (or $\tau_{cw} = \tau_{ccw}$) — 1 point

Summing torques about the right end of the rod:

$F_a\ell - Mg\left(\frac{\ell}{2}\right) = 0$, where F_a is force exerted by axis

$F_a = \frac{Ma}{2}$ — 1 point

F_a is directed upward — 1 point

(Alternate Solution for last two points) — (Alternate Points)

Sum torques about any other axis and also use $\sum F = 0$.

For example, summing torques about left end of rod:

$Mg\left(\frac{\ell}{2}\right) - F_t\ell = 0$, where F_t is force exerted by thread

$F_t = \frac{Mg}{2}$

$\sum F = 0$, so $F_t + F_a - Mg = 0$

$F_a = Mg - F_t = Mg - \frac{Mg}{2}$

$F_a = \frac{Mg}{2}$ — (1 point)

F_a is directed upward — (1 point)

(b) 2 points

Using $\sum\tau = I\alpha$ and calculating the torque about the axis end of the rod: — 1 point

$$Mg\ \frac{\ell}{2} = M\ \frac{\ell^2}{3}\ \alpha$$

Solving for α:

$$\alpha = \frac{3}{2}\ \frac{g}{\ell}$$ — 1 point

Mech 3. (continued)

(c) 2 points

Using the relation between translational and angular acceleration:

$a = \alpha r$ — 1 point

Substituting $r = \frac{\ell}{2}$ and α from previous part:

$a = \frac{3}{2}\frac{g}{\ell}\frac{\ell}{2}$

$a = \frac{3}{4}g$ — 1 point

(d) 3 points

Using Newton's Second Law:

$\sum F = Ma$ — 1 point

but $\sum F = Mg - F_r$ — 1 point

so $Mg - F_r = Ma$

Substituting $a = \frac{3}{4}g$ and solving for F_r:

$F_r = \frac{1}{4}Mg$ — 1 point

(e) 4 points

Using conservation of energy the increase in kinetic energy of rotation K_{rot} is equal to the decrease in potential energy ΔU — 1 point

$\Delta K_{rot} = \Delta U$

$\Delta K_{rot} = \frac{1}{2}I\omega^2$ — 1 point

$\Delta U = mgh$

$= Mg\frac{\ell}{2}\sin\theta$ — 1 point

$\frac{1}{2}I\omega^2 = Mg\frac{\ell}{2}\sin\theta$

Solving for ω:

$\omega = \sqrt{\frac{Mg\ell}{I}\sin\theta}$

Substituting $I = M\ell^2/3$

$\omega = \sqrt{\frac{3g}{\ell}\sin\theta}$ — 1 point

Distribution of Points

Mech 3. (continued)
(e) (continued)

(Alternate Solution) (Alternate points)

Work done by gravitional force W_g equals the increase in kinetic energy of rotation K_{rot} (1 point)

$$W_g = \int Mg \frac{\ell}{2} \cos\theta \, d\theta$$ (1 point)

$$= \frac{Mg\ell}{2} \sin\theta$$

$$\Delta K_{rot} = \frac{1}{2} I\omega^2$$ (1 point)

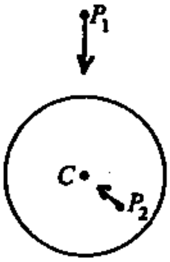
$$\frac{1}{2} I\omega^2 = Mg \frac{\ell}{2} \sin\theta$$

Substituting $I = M\ell^2/3$ and solving for ω:

$$\omega = \sqrt{\frac{3g}{\ell} \sin\theta}$$ (1 point)

E & M 1.

(a) 2 points



1 point for each correct vector — 2 points

(If both vectors are reversed from correct directions, then partial credit of 1 point awarded)

(b) i. 4 points

Gauss's Law:

$\oint \mathbf{E} \cdot d\mathbf{A} = Q_{encl}/\epsilon_0$ (or $4\pi k Q_{encl}$) — 1 point

For $r > R$, using a Gaussian surface that is a cylinder of radius r and length ℓ:

$\oint \mathbf{E} \cdot d\mathbf{A} = E(2\pi r \ell)$ — 1 point

$Q_{encl} = \rho(\pi R^2 \ell)$ — 1 point

$E(2\pi r \ell) = \rho(\pi R^2 \ell)/\epsilon_0$

$E = \frac{\rho R^2}{2\epsilon_0 r}$ $\left(\text{or } \frac{2\pi k \rho R^2}{r}\right)$ — 1 point

(b) ii. 2 points

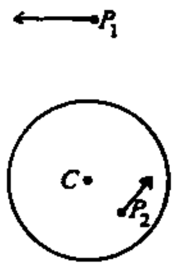
For $r < R$, using a similar Gaussian surface as above:

$E(2\pi r \ell) = \rho(\pi r^2 \ell)/\epsilon_0$ — 1 point

$E = \frac{\rho r}{2\epsilon_0}$ (or $2\pi k \rho r$) — 1 point

E & M 1. (continued)

(c) 3 points



1 point for first correct vector — 1 point

2 points for second correct vector — 2 points

(If vectors are reversed from correct directions or if circular lines of force with counterclockwise arrows shown, then partial credit of 1 point awarded.)

(d) 4 points

Ampere's Law:

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{encl}$, where I_{encl} is the current enclosed by the closed loop of integration (i.e. the current density times the area) — 1 point

For $r < R$, integrating over a loop of radius r:

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = B(2\pi r)$ — 1 point

$I_{encl} = \left(\frac{I}{\pi R^2}\right) \pi r^2 = I \frac{r^2}{R^2}$ — 1 point

$B(2\pi r) = \mu_0 I \frac{r^2}{R^2}$

$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2} \quad \left(\text{or } \frac{\mu_0 J r}{2} \right)$ — 1 point

E & M 2.

(a) i. 2 points

Using the expression for the flux of a uniform field:

$\Phi = \mathbf{B} \cdot \mathbf{A}$ — 1 point

Substituting:

$\Phi = abB_0$ — 1 point

(a) ii. 1 point

Using the expression relating the emf and flux:

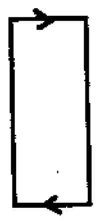
$\mathcal{E} = -\frac{d\Phi}{dt}$

Both the field and area are constant, so $\mathcal{E}$ = zero — 1 point

(a) iii. 1 point

Since there is no emf there is no current in the loop, and thus no magnetic force exerted on the loop. — 1 point

(b) 2 points



2 points

When $\omega t = \pi/2$, $B = B_0 \cos \pi/2$ = zero, i.e. the field has been decreasing, and is about to change direction. The induced current will be in a direction to oppose this change, i.e. clockwise.

(c) i. 4 points

Calculating the flux:

$\Phi = abB_0 \cos \omega t$ — 1 point

Calculating the emf:

$\mathcal{E} = -\frac{d\Phi}{dt}$ (negative sign not required) — 1 point

$= ab\omega B_0 \sin \omega t$

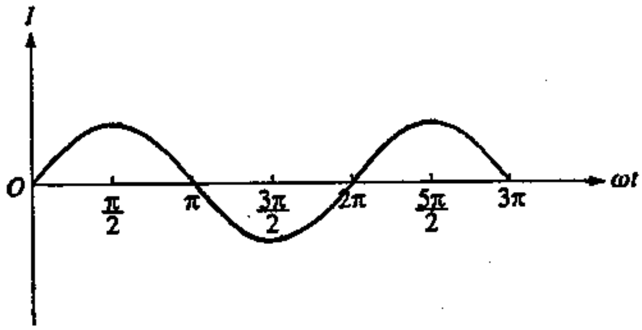
Using Ohm's Law:

$I = \mathcal{E}/R$ — 1 point

$I = \frac{ab\omega B_0}{R} \sin \omega t$ — 1 point

E & M 2. (continued)

(c) ii. 3 points



For a graph showing a period of 2π — 1 point

For a graph consistent with answer to (c)i. — 1 point

For correct orientation of current at $\pi/2$ consistent with answer to (b) (i.e., graph positive at $\pi/2$ if clockwise current in (b); graph negative at $\pi/2$ if counterclockwise current in (b)). — 1 point

(c) iii. 2 points

The maximum value of the current occurs when $\sin \omega t = 1$.

$$I_{max} = \frac{ab\omega B_0}{R}$$

For indicating the coefficient of the sin (or cos) term in (c)i. — 1 point

For the correct answer (i.e. the coefficient in (c)i. must be correct) — 1 point

Distribution of Points

E & M 3.

(a) 2 points

The force due to the magnetic field provides the centripetal force that causes the ion to move in the semicircle.

$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, so by the right-hand rule the magnetic field must point into the page (or in the $-z$ direction).

For field being perpendicular to the page — 1 point

For direction into the page or in the $-z$ direction — 1 point

(b) 1 point

Between the plates, the electric field must exert a force opposite to that of the magnetic field.

The magnetic force is to the right, and $\mathbf{F}_{elec} = q\mathbf{E}$, so the electric field should point toward the left.

Therefore, plate K should have a positive polarity with respect to plate L. — 1 point

(c) 2 points

Using the relation between the electric field and potential difference for parallel plates:

$E = V/d$ — 1 point

Substituting:

$E = (1500\ \text{V})/(0.012\ \text{m})$

$E = 1.25 \times 10^5\ \text{V/m}$ — 1 point

(d) 4 points

For a particle to pass between the plates undeflected, the forces due to the electric and magnetic fields F_E and F_B respectively must be equal in magnitude and in opposite directions. — 1 point

$F_E = qE$ — 1 point

$F_B = qvB$ — 1 point

Therefore, $qE = qvB$

Solving for v:

$v = E/B = (1.25 \times 10^5\ \text{V/m})/(0.20\ \text{T})$

$v = 6.25 \times 10^5\ \text{m/s}$ — 1 point

E & M 3. (continued)

(e) 3 points

The centripetal force F_c is equal to the force qvB due to the magnetic field. — 1 point

$F_c = \frac{mv^2}{R}$ — 1 point

$\frac{mv^2}{R} = qvB$

Solving for m:

$m = qBR/v$

Substituting:

$m = (1.6 \times 10^{-19}\ \text{C})(0.20\ \text{T})(0.50\ \text{m})/(6.25 \times 10^5\ \text{m/s})$

$m = 2.56 \times 10^{-26}$ kg — 1 point

(f) 2 points

Substituting $2q$ into the force equation from part (e) and solving for the new radius R': — 1 point

$R' = \frac{mv}{2qB}$

Substituting the expression for m from part (e):

$R' = \frac{v}{2qB} \; \frac{qBR}{v} = R/2$

$R' = 0.25$ m — 1 point

UNITS: Additional 1 point awarded if all units are correct — 1 point