Munkres - Topology - Part II Algebraic Topology
Ch 9 - The Fundamental Group

Defn Retraction

If $A \subset X$, a retraction of X onto A is a continuous map $r : X \to A$ such that $r|A$ is the identity map of A, I_A.

Lemma 55.1

If A is a retract of X, then the homomorphism of the fundamental groups induced by ~~the~~ (?) inclusion $j : A \to X$ is injective

Proof: If $r : X \to A$ is a retraction, then the composite map $r \circ j$ equals the identity map of A. It follows that $r_* \circ j_*$ is the identity map of $\pi_1(A, a)$, so that j_* must be injective.

*** PROBLEMS ***

— see next page.

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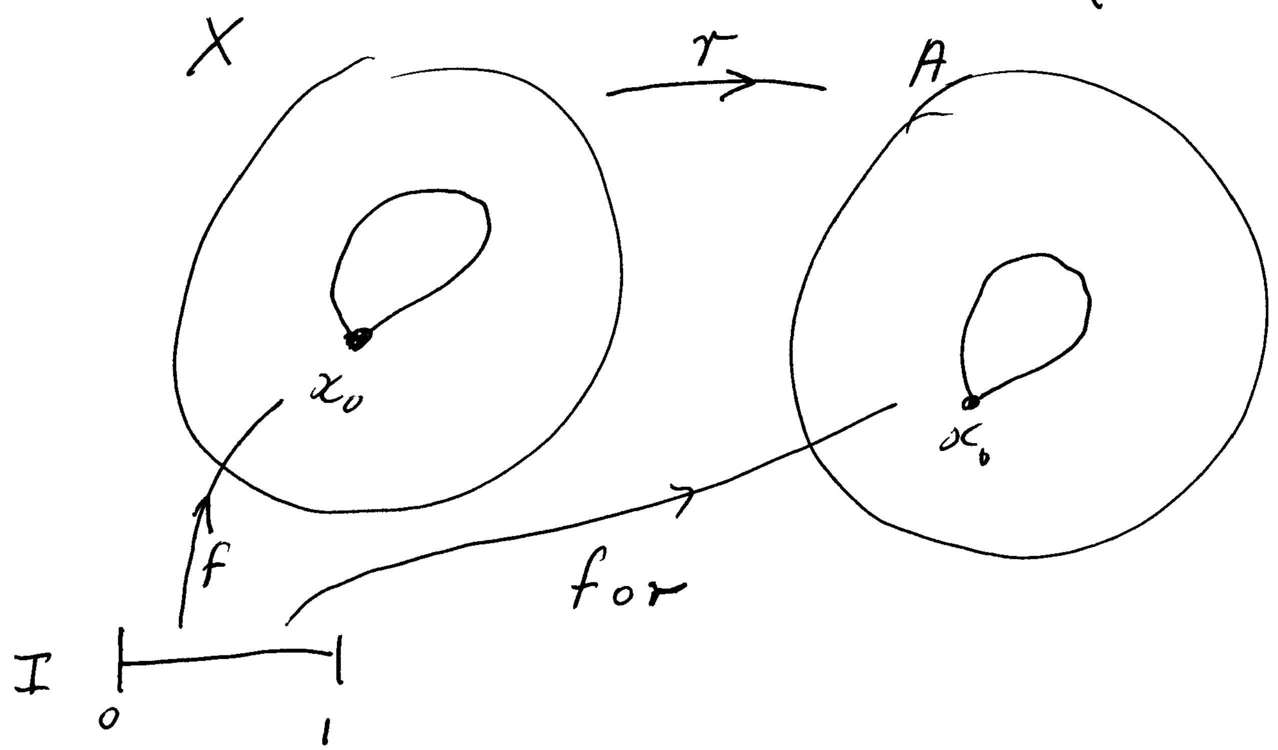
*** PROBLEMS ***

2. $r : X \to A$ when $r(x) = x$ for all $x \in A$

I am trying to understand the homomorphism induced by r, r_*

[I understand that $A \subset X$ but am showing ~~them~~ the sets A and X as separate for clarity - so can follow the def^n of h_* on page 333 of Munkres (see attachment)



Following Munkres def^n of h_* on Page 333 (see attachment) we require that $r : (X, x_0) \to (A, x_0)$ is a continuous map that carries the point x_0 of X to the point x_0 of A.

B

*** PROBLEMS *** (Continued)

(2) continued

f is a loop in X based at x_0

~~Bu~~ ~~what~~

BUT

* what if the loop f and (possibly anyway) x_0 are in X but outside of A ???

Anyway, continuing – given that f is a loop in X based at x_0, then the composite $r \circ f : I \to A$ is a loop in A based at x_0

Thus the correspondence $f \to r \circ f$ gives rise to a map carrying $\pi_1(X, x_0)$ into $\pi_1(A, x_0)$

ie $r_* : \pi_1(X, x_0) \to \pi_1(A, x_0)$

Is this correct so far?

(Continued next page)

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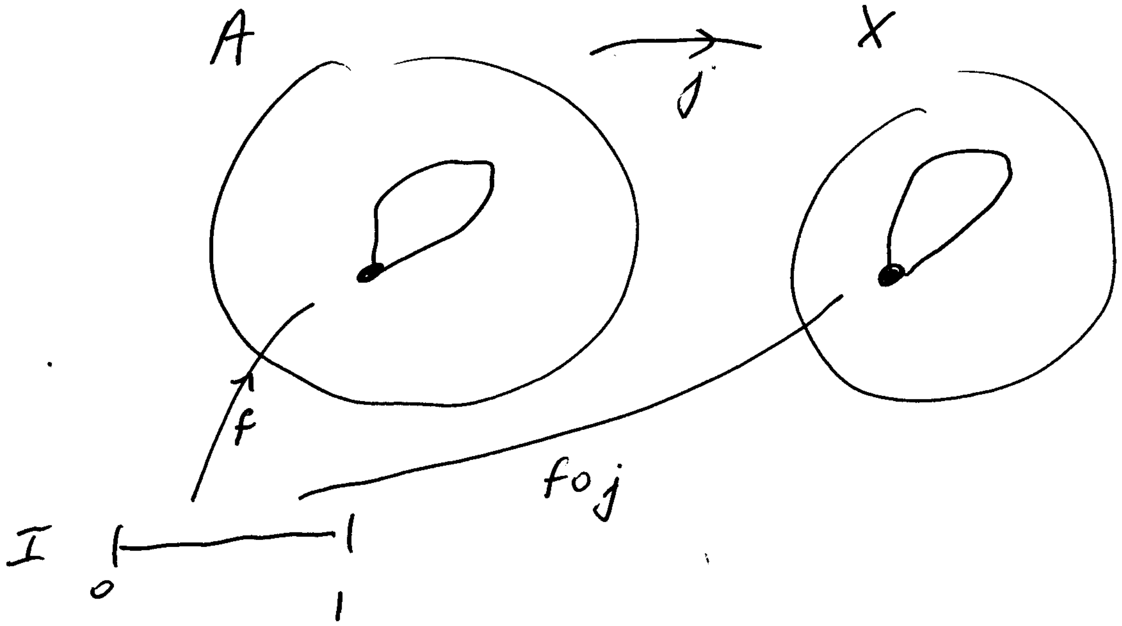
*** PROBLEMS ***

(3) Argument to form j_*

$j : A \to X$ where $j(a) = a$ for all $a \in A$.



By a similar argument to the situation with r and r_* we have

$$j_* : \pi_1(A, x_0) \to \pi_1(X, x_0)$$

But we have

$$r_* : \pi_1(X, x_0) \to \pi_1(A, x_0)$$

Then $r_0 \circ j_* : \pi_1(X, x_0) \to \pi_1(A, x_0) \to \pi_1(X, x_0)$

loops in X ~~start~~ with base point x_0

loops in A – base point x_0

loops in X, base point x_0

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*** Problems ***

(3) continued

Thus

loops in A, base point x_0 — loops in X, base points x_0 — loops in A, base point x_0

$$r_* \circ j_* : \pi_1(A, x_0) \longrightarrow \pi_1(X, x_0) \longrightarrow \pi_1(A, x_0)$$

ie

$$r_* \circ j_* : \pi_1(A, x_0) \longrightarrow \pi_1(A, x_0)$$

(Is all of the above correct???)

I ~~sd~~ suspect that $r_* \circ j_*$ is the identity map of $\pi_1(A, a)$ – intuitively it seems so – but how would you formally show this.

What is the ~~f~~ set of formal + explicit steps that show the following

" The composite map $r \circ j$ equals the identity map of A $\implies$ $r_* \circ j_*$ is the identity map of $\pi_1(A, x_0)$ "

Also, can someone confirm that my reasoning above is correct?