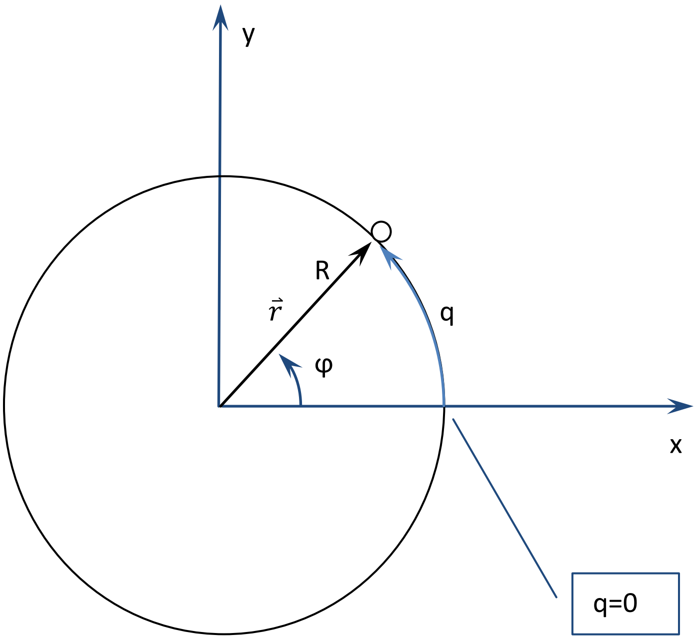

The radius equals R

$$|\vec{r}| = R$$

The q is the length of circle, and in radians can be written

$$q = R\,\varphi$$

(complete circle is q = 2 π R)

If the trajectory is circle than

$$\vec{r} = R\,(\hat{x}\cos\varphi + \hat{y}\sin\varphi)$$

or

$$\vec{r} = R\,\left(\hat{x}\cos\frac{q}{R} + \hat{y}\sin\frac{q}{R}\right)$$

and the derivative is

$$\frac{d\vec{r}}{dq} = R\,\left(-\hat{x}\frac{1}{R}\sin\frac{q}{R} + \hat{y}\frac{1}{R}\cos\frac{q}{R}\right) = -\hat{x}\sin\frac{q}{R} + \hat{y}\cos\frac{q}{R}$$

that is in the same direction as velocity on a circle.

Now, if force is radial and its origin is the same as the origin of the circle

$$\vec{f} = f(r)\ \left(\hat{x}\cos\frac{q}{R} + \hat{y}\sin\frac{q}{R}\right)$$

(the expression in the bracket is the unit vector in radial direction)

but then the total force on the body is zero because

$$m\,\ddot{q} = \frac{d\vec{r}}{dq}\cdot\vec{f} = \ \left(-\hat{x}\sin\frac{q}{R} + \hat{y}\cos\frac{q}{R}\right)\ \cdot\ f(r)\left(\hat{x}\cos\frac{q}{R} + \hat{y}\sin\frac{q}{R}\right)$$

$$m\,\ddot{q} = \frac{d\vec{r}}{dq}\cdot\vec{f} = \ f(r)\left(-\sin\frac{q}{R}\ \cos\frac{q}{R} + \cos\frac{q}{R}\ \sin\frac{q}{R}\right) = 0$$

from where

$$\ddot{q} = 0$$

$$\dot{q} = v$$

and

$$\vec{N} = -\vec{f} + m\,\ddot{q}\ \frac{d\vec{r}}{dq} + m\,\dot{q}^2\ \frac{d^2\vec{r}}{dq^2}$$

$$\vec{N} = -\vec{f} + m\,v^2\ \frac{d^2\vec{r}}{dq^2}$$

$$\vec{N} = -\vec{f} + m\,v^2\left(-\hat{x}\frac{1}{R}\cos\left(\frac{q}{R}\right) - \hat{y}\frac{1}{R}\sin\left(\frac{q}{R}\right)\right)$$

$$\vec{N} = -f(r)\left(\hat{x}\cos\frac{q}{R} + \hat{y}\sin\frac{q}{R}\right) + m\,v^2\left(-\hat{x}\frac{1}{R}\cos\left(\frac{q}{R}\right) - \hat{y}\frac{1}{R}\sin\left(\frac{q}{R}\right)\right)$$

$$\vec{N} = \ \left(-f(r) - \frac{m\,v^2}{R}\right)\left(\hat{x}\cos\frac{q}{R} + \hat{y}\sin\frac{q}{R}\right)$$

where the force is in radial direction toward origin and with magnitude

$$N = \ f(r) + \frac{m\,v^2}{R}$$