$$y(s,t) = v\left(x(s,t),t\right)$$

$$\left.\frac{\partial y}{\partial t}\right|_{s,t} = \left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial v}{\partial t}\right|_{x(s,t),t}$$

$$\left.\frac{\partial^2 y}{\partial t^2}\right|_{s,t} = \left.\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial t}\right)\right|_{s,t} = \frac{\left.\frac{\partial y}{\partial t}\right|_{s,t+dt} - \left.\frac{\partial y}{\partial t}\right|_{s,t}}{dt}$$

$$= \frac{\left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t+dt),t+dt} \left.\frac{\partial x}{\partial t}\right|_{s,t+dt} + \left.\frac{\partial v}{\partial t}\right|_{x(s,t+dt),t+dt}\right) - \left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial v}{\partial t}\right|_{x(s,t),t}\right)}{dt}$$

$$= \frac{\left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t)+\left.\frac{\partial x}{\partial t}\right|_{s,t} dt,t+dt} \left(\left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 x}{\partial t^2}\right|_{s,t} dt\right) + \left.\frac{\partial v}{\partial t}\right|_{x(s,t)+\left.\frac{\partial x}{\partial t}\right|_{s,t} dt,t+dt}\right) - \left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial v}{\partial t}\right|_{x(s,t),t}\right)}{dt}$$

$$= \frac{\left(\left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} + \left.\frac{\partial^2 v}{\partial x^2}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} dt + \left.\frac{\partial^2 v}{\partial x \partial t}\right|_{x(s,t),t} dt\right)\left(\left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 x}{\partial t^2}\right|_{s,t} dt\right) + \left(\left.\frac{\partial v}{\partial t}\right|_{x(s,t),t} + \left.\frac{\partial^2 v}{\partial x \partial t}\right|_{s,t} \left.\frac{\partial x}{\partial t}\right|_{s,t} dt + \left.\frac{\partial^2 v}{\partial t^2}\right|_{s,t} dt\right)\right) - \left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial v}{\partial t}\right|_{x(s,t),t}\right)}{dt}$$

$$= \frac{\left(\left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} + \left(\left.\frac{\partial^2 v}{\partial x^2}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 v}{\partial x \partial t}\right|_{x(s,t),t}\right) dt\right)\left(\left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 x}{\partial t^2}\right|_{s,t} dt\right) + \left(\left.\frac{\partial v}{\partial t}\right|_{x(s,t),t} + \left(\left.\frac{\partial^2 v}{\partial x \partial t}\right|_{s,t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 v}{\partial t^2}\right|_{s,t}\right) dt\right)\right) - \left(\left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial v}{\partial t}\right|_{x(s,t),t}\right)}{dt}$$

$$= \frac{\left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} \left.\frac{\partial^2 x}{\partial t^2}\right|_{s,t} dt + \left(\left.\frac{\partial^2 v}{\partial x^2}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 v}{\partial x \partial t}\right|_{x(s,t),t}\right) \left.\frac{\partial x}{\partial t}\right|_{s,t} dt + \left(\left.\frac{\partial^2 v}{\partial x \partial t}\right|_{s,t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 v}{\partial t^2}\right|_{s,t}\right) dt}{dt}$$

$$= \left.\frac{\partial v}{\partial x}\right|_{x(s,t),t} \left.\frac{\partial^2 x}{\partial t^2}\right|_{s,t} + \left(\left.\frac{\partial^2 v}{\partial x^2}\right|_{x(s,t),t} \left.\frac{\partial x}{\partial t}\right|_{s,t} + 2\left.\frac{\partial^2 v}{\partial x \partial t}\right|_{x(s,t),t}\right) \left.\frac{\partial x}{\partial t}\right|_{s,t} + \left.\frac{\partial^2 v}{\partial t^2}\right|_{s,t}$$