

we have  $\mathcal{E}_k(\eta_0) = \omega$ ,  $\mathcal{E}_k(\eta_0) = \omega$ .

However, the evolution equations (2.11) drive  $F_k$  different from zero and a diagonal form for  $H$  can be recovered only after a Bogolyubov transformation on the creation/annihilation operators

$$\begin{aligned}\hat{a}(\eta, k) &\equiv \alpha_k(\eta)a(k) + \beta_k(\eta)b^+(-k), \\ \hat{b}^+(\eta, k) &\equiv -\beta_k^*(\eta)a(k) + \alpha_k^*(\eta)b^+(-k).\end{aligned}\quad (2.16)$$

While it is immediate to see that the equal time anticommutation relations on both sets  $\{a^{(+)}, b^{(+)}\}$  and  $\{\hat{a}^{(+)}, \hat{b}^{(+)}\}$  enforce

$$|\alpha_k|^2 + |\beta_k|^2 = 1, \quad \boxed{\hspace{2cm}} \quad (2.17)$$

the use of some algebra shows that a diagonalization of the hamiltonian is possible with the choice

$$\alpha = \beta \left( \frac{E + \omega}{F^*} \right), \quad |\beta|^2 = \frac{|F|^2}{2\omega(E + \omega)} = \frac{\omega - E}{2\omega}. \quad (2.18)$$

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<sup>4</sup>We do not deal with the analogous equations for  $v_{\pm}$ , since from condition (2.7) it simply follows  $v_+(\eta) = -u_-^*(\eta)$ ,  $v_-(\eta) = u_+^*(\eta)$ .

The time varying operators  $\hat{a}^{(+)}$  and  $\hat{b}^{(+)}$  are employed to define time dependent Fock spaces, each of them built from the zero (quasi)particle state at the time  $\eta$ :

$$\hat{a}(\eta)|0_{\eta}\rangle = \hat{b}(\eta)|0_{\eta}\rangle = 0. \quad (2.19)$$

Let us consider the state  $|0_{\eta_0}\rangle \equiv |0\rangle$  which describes a system with zero particles at the initial time  $\eta_0$ . This is no longer true for generic time  $\eta > \eta_0$ , since the occupation number at  $\eta$  is defined in terms of the operators which diagonalize the hamiltonian at that moment. The particle density per physical volume  $V = a^3$  at time  $\eta$  is indeed given by<sup>5</sup>

$$\begin{aligned}n(\eta) &\equiv \langle 0 | \frac{N}{V} | 0 \rangle = \langle 0 | \frac{1}{a^3} \sum_{r=\pm 1} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \hat{a}_r^+(\eta, r) \hat{a}_r(\eta, r) | 0 \rangle = \\ &= \frac{1}{\pi^2 a^3} \int dk k^2 |\beta_k|^2, \quad \boxed{\hspace{2cm}}\end{aligned}\quad (2.20)$$

which is different from zero whenever the hamiltonian is non diagonal in terms of the initial operators. The occupation number of created fermions is thus given by  $n_k = |\beta_k|^2$  and the condition (2.17) ensures that the Pauli limit  $n_k < 1$  is respected.