



We can write \mathbf{Q}' in terms of \mathbf{Q} as approximately

$$\begin{aligned} \mathbf{Q}' &\approx \mathbf{Q} + (Q \sin \alpha) \delta\theta (\text{in direction of } \hat{\mathbf{n}} \times \mathbf{Q}) \\ &\approx \mathbf{Q} + |\hat{\mathbf{n}} \times \mathbf{Q}| (\text{in direction of } \hat{\mathbf{n}} \times \mathbf{Q}) \delta\theta \\ &\approx \mathbf{Q} + \hat{\mathbf{n}} \times \mathbf{Q} \delta\theta \\ &\approx \mathbf{Q} + \delta\boldsymbol{\theta} \times \mathbf{Q} \end{aligned}$$

This means that we can express the derivative of this vector as:

$$\begin{aligned} \frac{\mathbf{Q}' - \mathbf{Q}}{dt} &= \frac{\delta\boldsymbol{\theta} \times \mathbf{Q}}{dt} \\ \frac{d\mathbf{Q}}{dt} &= \frac{\delta\boldsymbol{\theta}}{dt} \times \mathbf{Q} \end{aligned}$$

and so we have:

$$\frac{d\mathbf{Q}}{dt} = \boldsymbol{\Omega} \times \mathbf{Q} \quad \text{rate of change of vector rotating at constant rate } \boldsymbol{\omega}$$