This is what I was thinking.

$$\omega_n = 2\omega_0 \sin \frac{k_n a}{2} \tag{1}$$

$$k_n = \frac{n\pi}{(N+1)a} \tag{2}$$

$$\frac{k_n a}{2} = \frac{n\pi}{(N+1)a}\frac{a}{2} = \frac{n\pi}{(N+1)2} \tag{3}$$

There in the approximated sine where the continuum limit is applied the relationship for ω_n becomes

$$\omega_n = 2\omega_0 \frac{n\pi}{2(N+1)} \tag{4}$$

Where $n << N$ in order to approximate the sine with the taylor series. Therefore $n > N$ or $n \approx N$ is not valid.