Statement:

For $n \geq 2$ and $0 < k < n$,

$$P(n) = P(k)P(n-k) - P(k-1)P(n-k-1)$$

Proof (by induction on k):

- Initial step:

  Since $P(1) = x$ and $P(0) = 1$, the recurrence rule for $P(n)$ can be transformed into

  $$\begin{aligned} P(n) &= xP(n-1) - P(n-2) && (1) \\ &= P(1)P(n-1) - P(0)P(n-2) && (2) \end{aligned}$$

  which is the desired statement for $k = 1$.

- Induction step:

  Assume the statement true for $k = m$. Then, for $k = m+1$ (as long as $k < n$),

  $$\begin{aligned} &P(m+1)P(n-(m+1)) - P((m+1)-1)P(n-(m+1)-1) \\ &\quad = P(m+1)P(n-m-1) - P(m)P(n-m-2) && (3) \\ &\quad = (xP(m) - P(m-1))\,P(n-m-1) - P(m)P(n-m-2) && (4) \\ &\quad = xP(m)P(n-m-1) - P(m-1)P(n-m-1) - P(m)P(n-m-2) && (5) \\ &\quad = P(m)\,(xP(n-m-1) - P(n-m-2)) - P(m-1)P(n-m-1) && (6) \\ &\quad = P(m)P(n-m) - P(m-1)P(n-m-1) && (7) \\ &\quad = P(n) && (8) \end{aligned}$$

  by the induction hypothesis.