

From the substitution:

$$y = -\frac{1}{f} \frac{df}{dx}$$

We get

$$\frac{d^2 f}{dx^2} + x^2 f = 0$$

Which is solved to obtain:

$$f(x) = \sqrt{x} \left[ A J_{\frac{1}{4}} \left( \frac{x^2}{2} \right) + B J_{-\frac{1}{4}} \left( \frac{x^2}{2} \right) \right]$$

From the relation  $J_{-\nu}(z) = (-1)^\nu J_\nu(z)$  we write:

$$f(x) = \left( A + (-1)^{\frac{1}{4}} B \right) \sqrt{x} J_{\frac{1}{4}} \left( \frac{x^2}{2} \right) \text{ OR } f(x) = C \sqrt{x} J_{\frac{1}{4}} \left( \frac{x^2}{2} \right)$$

$$\Rightarrow \frac{df}{dx} = C \left\{ \frac{1}{2\sqrt{x}} J_{\frac{1}{4}} \left( \frac{x^2}{2} \right) + x\sqrt{x} \left[ \frac{1}{2x^2} J_{\frac{1}{4}} \left( \frac{x^2}{2} \right) - J_{\frac{5}{4}} \left( \frac{x^2}{2} \right) \right] \right\}$$

Having used  $J'_\nu(z) = \frac{\nu}{z} J_\nu(z) - J_{\nu+1}(z)$

$$\therefore y = -\frac{1}{f} \frac{df}{dx} = -\frac{C \left\{ \frac{1}{2\sqrt{x}} J_{\frac{1}{4}} \left( \frac{x^2}{2} \right) + x\sqrt{x} \left[ \frac{1}{2x^2} J_{\frac{1}{4}} \left( \frac{x^2}{2} \right) - J_{\frac{5}{4}} \left( \frac{x^2}{2} \right) \right] \right\}}{C \sqrt{x} J_{\frac{1}{4}} \left( \frac{x^2}{2} \right)}$$

$$= \frac{x J_{\frac{5}{4}} \left( \frac{x^2}{2} \right)}{J_{\frac{1}{4}} \left( \frac{x^2}{2} \right)} - \frac{1}{x} + D; D \text{ is a constant of integration}$$

$$\text{From the initial condition, } y(1) = 0, \text{ we obtain } D = 1 - \frac{J_{\frac{5}{4}} \left( \frac{1}{2} \right)}{J_{\frac{1}{4}} \left( \frac{1}{2} \right)}$$

With the above relations  $y(2) = 5.393687941$ . But from the Runge–Kutta–Fehlberg, I had  $y(2) = 6.7479267$ .