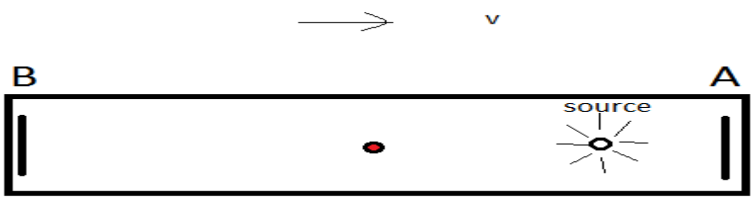
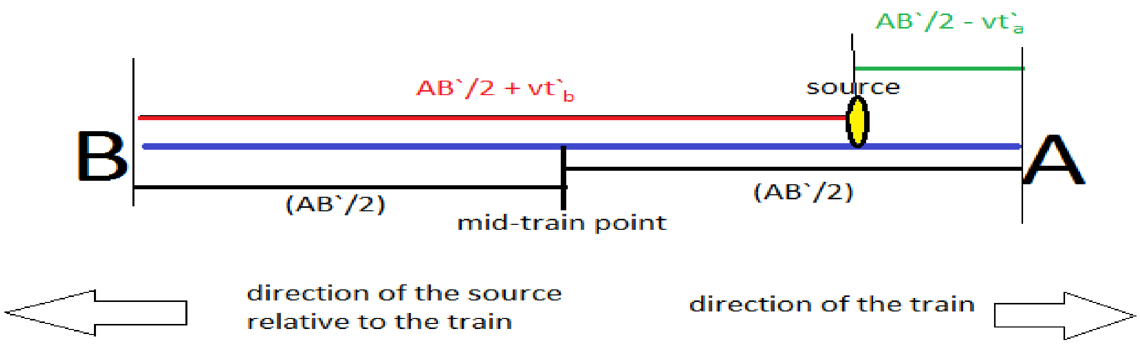
Let's assume there is a train moves toward the right direction of the page. There are 2 ends A to the right and B to the left as shown in the picture. A light source is put near A than B and is emitting 2 beams of light so as to appear reaching both ends simultaneously for a ground observer. That will happen when the source coincides with the iso-center point of the train



For the train observer, he sees the source moves from the right to the left side and I am interested to calculate the time that received by 2 observers at A & B of the train



$$t`_a = \frac{\left(\frac{1}{2}AB`\right) - vt`a}{c} \quad \text{and} \quad t`_b = \frac{\left(\frac{1}{2}AB`\right) + vt`b}{c}$$

$$t`_a = \frac{(\frac{1}{2}AB`)}{c} - \frac{v}{c}\, t`_a \quad \text{and} \quad t`_b = \frac{(\frac{1}{2}AB`)}{c} + \frac{v}{c}\, t`_b$$

$$t`_a\,(1+\frac{v}{c}) = \frac{(\frac{1}{2}AB`)}{c} \quad \text{and} \quad t`_b\,(1-\frac{v}{c}) = \frac{(\frac{1}{2}AB`)}{c}$$

$$t`_a = \frac{(\frac{1}{2}AB`/c)}{1+\frac{v}{c}} \quad \text{and} \quad t`_b = \frac{(\frac{1}{2}AB`/c)}{1-\frac{v}{c}}$$

$$\Delta t` = t`_b - t`_a = \frac{(\frac{1}{2}AB`/c)}{1-\frac{v}{c}} - \frac{(\frac{1}{2}AB`/c)}{1+\frac{v}{c}}$$

$$= (\frac{AB`}{2c}) \frac{1+\frac{v}{c}-1+\frac{v}{c}}{1-(\frac{v2}{c2})}$$

$$= (\frac{AB`}{2c}) \frac{2\frac{v}{c}}{1-(\frac{v2}{c2})}$$

$$\Delta t` = \frac{AB`\frac{v}{c2}}{1-(\frac{v2}{c2})} \qquad ,,,,,,,,,, (1)$$

At this stage I can use the length contraction formula $AB` = AB\sqrt{1-\frac{v2}{c2}}$ to give:

$\Delta t` = \frac{AB\frac{v}{c2}}{\sqrt{1-\frac{v2}{c2}}}$ which is Lorentz transformation

But it is enough for me to have just: $\Delta t` = \frac{AB`\frac{v}{c2}}{1-(\frac{v2}{c2})}$

Now the second part of the experiment is:

If A would send a light signal to B that will take : $\Delta t`` = \frac{AB`}{c}$,,,,,,,,,, (2)

Then $\Delta t` = \Delta t``$ when $\frac{AB`\frac{v}{c2}}{1-(\frac{v2}{c2})} = \frac{AB`}{c}$

That is when $\frac{\frac{v}{c}}{1-(\frac{v2}{c2})} = 1$

So $\frac{v2}{c2} + \frac{v}{c} - 1 = 0$

The positive root of this quadratic equation is when $\frac{v}{c}$ = 0.618