The Speed of Light

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Abstract

In this experiment, a beam of light from a laser was reflected off of a rotating mirror to a fixed mirror, and then reflected back to the rotating mirror. The returning light was focused to a point image in a microscope. Due to the continued rotation of the mirror while the light was in transit from the rotating mirror, to the fixed mirror, and back, the beam was reflected into the microscope at an altered angle, resulting in a displacement of the point image. Measurement of this displacement, along with distance parameters from our setup, known as the Foucault Method, allows for the speed of the light to be calculated. The mean speed of light calculated from this experiment was $(2.9972 \pm 0.0139) \times 10^8$ m/s.

Methodology

In this experiment, a beam of light from a laser travels through a series of lenses (L1 and L2) to reach a rotating mirror (RM). The light is reflected off of the rotating mirror to a fixed mirror (FM), and then reflected back to the rotating mirror. The rotating mirror directs the light into a beam splitter so that a point image can be seen in a microscope. This setup, used to measure the speed of light, is called the Foucault Method (see Figure 1). The variables A, B, and D represent distances within the setup, where

A = the distance between L1 and L2 (minus the focal length of L1)
B = the distance between L2 and RM
D = the distance between RM and FM

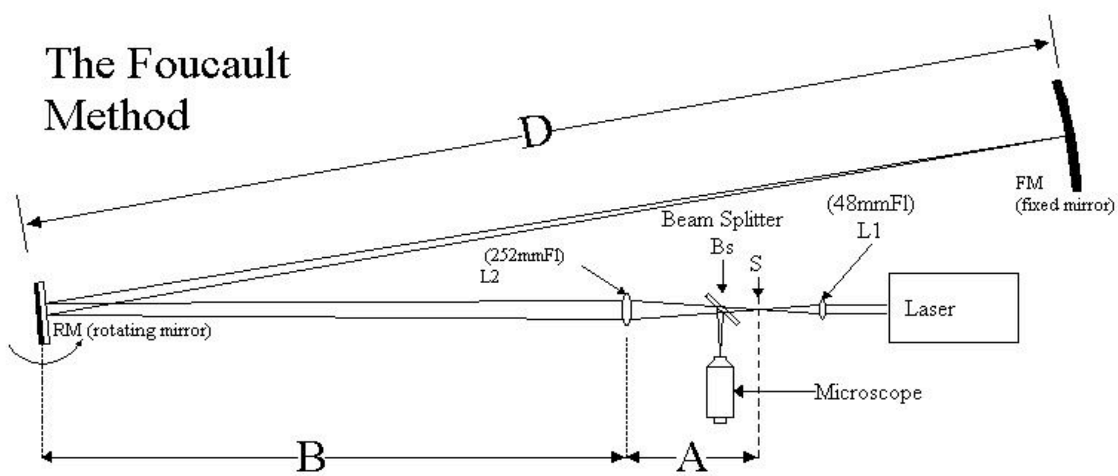

Figure 1: Diagram of Apparatus – The Foucault Method
("Measuring the speed")

The speed of light is determined by measuring the displacement of the point image in the microscope. The light originally hits the rotating mirror and is reflected to the fixed mirror at an angle of θ with respect to its original path. In the time that it takes the light to reach the fixed mirror and be reflected back to the rotating mirror, the rotating mirror has rotated slightly, changing the angle by $\Delta\theta$. This results in the point image being displaced from its original position, s, in the microscope, to a position of s'.

Once the laser, lenses, and rotating mirror were aligned linearly, the laser was turned on so that the beam could be reflected to the fixed mirror. The fixed mirror was placed a set distance from the rotating mirror, and also at an angle from the linear alignment, so that the light from the rotating mirror would hit the fixed mirror rather than internal components of the setup. The fixed mirror was then adjusted in position so that the beam was redirected along its original path to the rotating mirror, and a point image was seen in the microscope.

With the setup completely aligned, and the original position of the point image recorded, the rotating mirror was turned on to rotate with a high angular velocity in the clockwise direction. Due to the change in angle of the rotating mirror during the time that the light was en route to the fixed mirror and back, the point image was displaced by Δs in the microscope. This displacement was measured using a micrometer, and recorded. This procedure was repeated with the mirror rotating in the counterclockwise direction as well (Lee, 1989).

Data

The parameters of our setup remained constant throughout each trial. These variables (see Figure 1), used to calculate the speed of light, are

$$A = 0.26 \text{ m} \pm 0.0005 \text{ m}$$
$$B = 0.49 \text{ m} \pm 0.0005 \text{ m}$$
$$D = 11.8237 \text{ m} \pm 0.0500 \text{ m}$$

The procedure was conducted ten times for precision. For each trial, the clockwise rotational velocity (ω_{CW}), counterclockwise rotational velocity (ω_{CCW}), displacement from clockwise rotation (s_{CW}') and displacement from counterclockwise rotation (s_{CCW}') were recorded (Table 1). The raw data from Table 1 was condensed in Table 2, which shows the total rotational velocity and total image displacement by combining the values from the clockwise and counterclockwise directions.

$$\omega_{CW} + \omega_{CCW} = \Delta\omega_{total} \tag{1}$$

$$s'_{CW} - s'_{CCW} = \Delta s'_{total} \tag{2}$$

Table 1: Data Collection of Angular Velocity and Displacement				
Trial #	ω_{CW} (rev/s)	ω_{CCW} (rev/s)	s_{CW}' (m)	s_{CCW}' (m)
1	1498 ± 1	1489 ± 1	0.00037 ±0.00001	-0.000389 ± 0.00001
2	1501 ± 1	1488 ± 1	0.00034 ± 0.00001	-0.00037 ± 0.00001
3	1503 ± 1	1488 ± 1	0.000345 ± 0.00001	-0.00039 ± 0.00001
4	1502 ± 1	1491 ± 1	0.000395 ± 0.00001	-0.00037 ± 0.00001
5	1503 ± 1	1490 ± 1	0.00034 ± 0.00001	-0.00036 ± 0.00001
6	1504 ± 1	1489 ± 1	0.000395 ± 0.00001	-0.000345 ± 0.00001
7	1499 ± 1	1485 ± 1	0.00039 ± 0.00001	-0.00039 ± 0.00001
8	1498 ± 1	1486 ± 1	0.000385 ± 0.00001	-0.000355 ± 0.00001
9	1501 ± 1	1485 ± 1	0.00040 ± 0.00001	-0.000360 ± 0.00001
10	1498 ± 1	1486 ± 1	0.00038 ± 0.00001	-0.000360 ± 0.00001

Table 2: Summative Data from Table 1		
Trial #	$\Delta\omega_{total}$ (rev/s)	$\Delta s'_{total}$ (m)
1	2987 ± 2	0.000759 ± 0.00002
2	2989 ± 2	0.000710 ± 0.00002
3	2991 ± 2	0.000735 ± 0.00002
4	2993 ± 2	0.000765 ± 0.00002
5	2993 ± 2	0.000700 ± 0.00002
6	2993 ± 2	0.000740 ± 0.00002
7	2984 ± 2	0.000780 ± 0.00002
8	2984 ± 2	0.000740 ± 0.00002
9	2986 ± 2	0.000760 ± 0.00002
10	2984 ± 2	0.000740 ± 0.00002

Data Analysis

When the beam of light (incident ray) first strikes the rotating mirror, RM is at an angle θ with respect to the incident beam. The ray is then reflected at an additional angle of θ, making the angle between the incident ray and the reflected ray 2θ.

Light that leaves the laser later will reach the rotating mirror later into its rotation. The rotating mirror is now at an angle of θ', where

$$\theta' = \theta + \Delta\theta \tag{3}$$

Therefore, the reflected ray will have an angle of

$$2\theta' = 2(\theta + \Delta\theta) \tag{4}$$

The change in position of the beam on the fixed mirror, from S and S′, or the displacement (ΔS), can be calculated as

$$\Delta S = S' - S = D\,(2\theta' - 2\theta) = D[2(\theta + \Delta\theta) - 2\theta] = 2D\Delta\theta \tag{5}$$

The displacement of the point image, Δs, can be calculated from lens theory,

$$s' - s = \Delta s = (i/o)\,\Delta S \tag{6}$$

where *i* is the distance from the image to L2, o is the distance from the object (on fixed mirror) to L2, and ΔS is the displacement of the virtual image on the fixed mirror. This equation can be rewritten using the parameters of this experiment as

$$\Delta s' = \Delta s = \frac{i}{o}\Delta S = \frac{A}{D+B}\Delta S \tag{7}$$

Combining equations (5) and (7) results in an expression for the displacement of the point image in terms of the distances between components of the setup.

$$\Delta s' = \frac{2DA\Delta\theta}{D+B} \tag{8}$$

The change in the angle of the rotating mirror depends upon the angular velocity of mirror (f), measured in radians per second, and the speed of light (c). Higher angular speeds will result in a greater change in angle. The light travels to the fixed mirror and back to the rotating mirror, a distance of 2D, in the time that the rotating mirror rotates Δθ.

$$\Delta\theta = \frac{Df}{c} \tag{9}$$

By replacing Δθ in equation (8) with the expression for Δθ found in equation (9), the new equation utilizes experimentally measurable variables to determine the displacement of the point image.

$$\Delta s' = \frac{4AD^2 f}{c(D+B)} \tag{10}$$

Rearranging equation (10) to solve for the speed of light results in the equation,

$$c = \frac{4AD^2 f}{(D+B)\Delta s'} \tag{11}$$

By multiplying the angular velocity, f, originally in rad/s, by 2π, rotational velocity (ω) can be represented in revolutions per second (rev/s). The measurements of rotational velocity (ω) were taken both in the clockwise (ω_{CW}) and counterclockwise (ω_{CCW}) directions to minimize error. This resulted in a positive displacement (s'_{CW}) when the mirror was rotated clockwise, and a negative displacement (s'_{CCW}) when the mirror was rotated counterclockwise. The final calculations of the speed of light (c) use the total displacement of the point image ($\Delta s'_{total}$) and the total combined rotational velocity ($\Delta\omega_{total}$), which can be found in Table 2.

Incorporating these modifications into equation (11) results in our final equation from which the speed of light can be measured (Lee, 1989).

$$c = \frac{8\pi A D^2 (\Delta\omega_{total})}{(D+B)(\Delta s'_{total})} \qquad (12)$$

Using the data from Table 2 in equation (7), the speed of light can be calculated for each trial (Table 3).

Table 3: Calculated Speed of Light from Data in Table 2		
Trial #	c (10^8 m/s)	Error (10^8 m/s)
1	2.918633	± 0.042069
2	3.123194	± 0.047610
3	3.018982	± 0.044690
4	2.902530	± 0.041548
5	3.172051	± 0.048944
6	3.000588	± 0.044164
7	2.838152	± 0.039977
8	2.991566	± 0.044032
9	2.914793	± 0.041953
10	3.091819	± 0.044032

Error Analysis

Each individual variable had an error associated with it. This was accounted for using propagation of error. Values can be found in Table 3.

$$dc = \sqrt{\frac{dc}{dA}^2 dA^2 + \frac{dc}{dB}^2 dB^2 + \frac{dc}{dD}^2 dD^2 + \frac{dc}{d\omega}^2 d\omega^2 + \frac{dc}{ds'}^2 ds'^2}$$

Standard deviation of the mean was calculated using:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{i=n}(dc_i)^2}$$

where n represents the number of data sets, and dc_i is the individualized error for each data set. Through error analysis, our final result for the speed of light is (2.9972 ± 0.0139) x 10^8 m/s.

Conclusion

By means of the Foucault Method, the measured speed of light was calculated to be (2.9972 ± 0.0139) x 10^8 m/s. The speed of light in a vacuum is known to be precisely 2.99792458 x 10^8 m/s (Gibbs, 1996). However, because the experiment was not conducted in a vacuum, the index of refraction of the medium, which in this case was air, needs to be accounted for. The adjusted speed of light can be calculated using

$$v = \frac{c}{n}$$

where n is the index of refraction, c is the known speed of light, and v is the adjusted speed of light due to the medium. The index of refraction of light in air at 15 degrees Celsius is n = 1.00027712 (Elert). Therefore, the adjusted speed of light can be determined as

$$v = \frac{c}{n} = \frac{2.99792458 \times 10^8 \ {}^m/_s}{1.00027712} = 2.997094025 \times 10^8 \ {}^m/_s$$

In comparing the measured and actual values of the speed of light, the percent error was found to be 0.0035%. This error could be further diminished by increasing the displacement of the point image in the microscope for more accurate measurements. This could be accomplished by increasing the angular velocity at which the rotating mirror rotates, which would cause the angle of the mirror to change more during the light's path to the fixed mirror and back. Another means of increasing the displacement of the point image and decreasing error would be to increase the distance between the rotating mirror and the fixed mirror. The longer path of the light would provide more time for the mirror to rotate, resulting in an increased displacement.

References

Elert, G. *The physics hypertextbook*. Retrieved from http://physics.info/refraction/

Gibbs, P. (1996). *Is the speed of light constant?*. Retrieved from http://math.ucr.edu/home/baez/physics/Relativity/SpeedOfLight/speed_of_light.html

Lee, B. (1989). Speed of light apparatus. In D. Griffith (Ed.), *Instruction manual and experiment guide*. Roseville, CA: PASCO.

Measuring the speed of light using the foucault method. (n.d.). Retrieved from http://www.warren-wilson.edu/~physics/physics2/SpeedLight/SPEEDC.html