

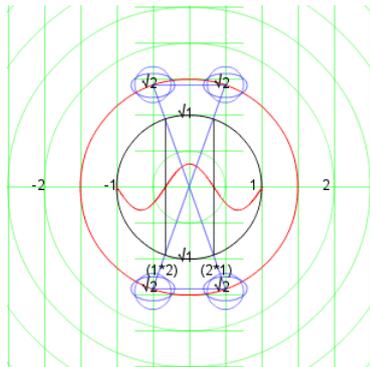
As you increment d from 1, 2, 3, ... onwards, you do two things:

- Draw concentric circles, whose radii begin at 1 and increment by $\frac{1}{2}$. In other words, on each turn the circle's diameter equals $d + 1$.
- Draw a sinusoid inside the unit circle, with increasing frequency.

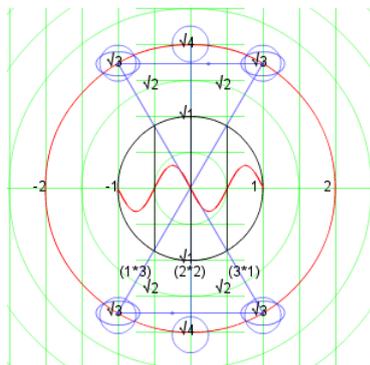
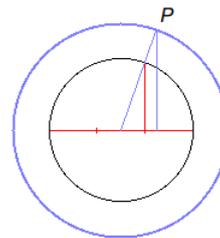
Now, please notice that you are not really using anything from the sinusoid, other than its zero (x -axis) intersects. That is, the sinusoid is a pretty artifact used **only** to split the diameter of the unit circle ($= 2$) in $d + 1$ equal parts, each of length $\frac{2}{d+1}$.

Of these equally spaced marks on the unit circle, the next-to-last is chosen (the one which is one space from the right end of the unit circle), in order to draw a vertical line from it. Since this mark is at a distance of $\frac{2}{d+1}$ (the space between marks) from the right end of the diameter (which is at $x = 1$), the mark (and the vertical line) has x coordinate $= 1 - \frac{2}{d+1}$.

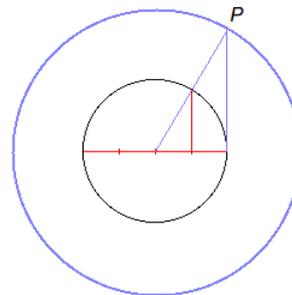
Here are a few screenshots of your Flash application for the cases $d = 2$ and $d = 3$, together with a simplified diagram of the distances involved.



Unit diameter divided in 3 parts



Unit diameter divided in 4 parts



In each case, the intriguing feature is that the height of the point marked as P on the outer circle has a height (y -coordinate) which appears to be exactly the square root of d . Here is an explanation of why this occurs.

The simplified figures on the right show two right triangles, sharing a vertex (at the origin) and the angle at that vertex. For a circle of radius R , any point of the

circle has coordinates

$$x = R \cos(\text{angle})$$

$$y = R \sin(\text{angle})$$

and, as the *angle* progresses, the point traces the circle. On the unit circle (where $R = 1$), the coordinates are simply the cosine and sine of the sweeping angle.

We know the x -coordinate of the point on the inner circle ($x = 1 - \frac{2}{d+1}$); thus the *angle* is the arc-cosine of that value of x .

Now, the outer circle has diameter $d + 1$ and therefore radius $\frac{d+1}{2}$, and so the height of P is

$$\begin{aligned} P_y &= \left(\frac{d+1}{2}\right) \sin(\text{angle}) \\ &= \left(\frac{d+1}{2}\right) \sin\left(\arccos\left(1 - \frac{2}{d+1}\right)\right) \end{aligned}$$

and, since

$$\sin(\arccos(x)) = \sqrt{1 - x^2}$$

the height of P simplifies to

$$\begin{aligned} P_y &= \left(\frac{d+1}{2}\right) \sqrt{1 - \left(1 - \frac{2}{d+1}\right)^2} \\ &= \left(\frac{d+1}{2}\right) \sqrt{1 - \left(1^2 - 2 \frac{2}{d+1} + \frac{2^2}{(d+1)^2}\right)} \\ &= \left(\frac{d+1}{2}\right) \sqrt{\frac{4}{d+1} - \frac{4}{(d+1)^2}} \\ &= \sqrt{\frac{4(d+1)^2}{4(d+1)} - \frac{4(d+1)^2}{4(d+1)^2}} \\ &= \sqrt{d+1 - 1} = \sqrt{d} \end{aligned}$$

as expected.

Please note that the formula for the height of P is **not** $\tan(\arccos(\dots))$; the tangent of the angle is represented, on the unit circle, by a line (drawn below in green) departing from the exact end of the diameter (and, appropriately, tangent to the unit circle); this vertical line only coincides with P in the case where the outer circle has radius 2 (because $\tan(\arccos(\frac{1}{2}))$ happens to be $\sqrt{3}$), but not in the general case.

