Effect of Local Conditions on the Flight Trajectory of an Indoor Badminton Shuttlecock

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1 Abstract

Local conditions such as temperature, humidity, and geographical altitude of a place directly affect the density of air, thus affecting air resistance and hence the flight of an indoor badminton shuttlecock. The effect of local conditions on the most common types of shots are analyzed. Even though the flight of the shuttlecock is quite a complex phenomenon, a few simplifying assumptions are made, notably the non-consideration of physical deformation of the shuttlecock especially just after racket impact. Results for two different conditions are presented – the first being an air-conditioned badminton court at sea level, and the second being a non-air conditioned court in Bangalore, which has an altitude of about 3000 feet. For the same shot played, it is shown that there is over 10% variation in the distance traveled by the shuttlecock across the conditions. It also requires about 40% more energy to be imparted to the shuttlecock in the first case (sea level) for it to follow a trajectory similar to that in the second case (high altitude).

2 Introduction

Indoor badminton is a racquet sport played with a feathered projectile called a shuttlecock. The shuttlecock is aerodynamically shaped with a conically arranged set of bird feathers anchored in a cork base. A shuttlecock weighs around 4.75 - 5.50 grams, mostly concentrated in the cork base. It has 14-16 feathers with each feather 70mm in length. The diameter of the cork is 25-28mm and the diameter of the circle that the feathers make is around 54mm. The shuttlecock is very light compared to the area it presents during travel because of which the effect of air resistance on its flight is very pronounced. When struck, the shuttlecock travels cork first, followed by the flexible feathered section. It rapidly loses speed as it travels through the air. An important consequence of its aerodynamic properties is that its fall is significantly steeper than its rise. The game has a lot of subtle strokes and deception making it imperative for a good player to understand the nuances of the flight of the shuttlecock in order to optimize the efficacy of a stroke. This nature of the game requires shuttlecocks to be very manufactured with a strict level of consistency else even the best of players can get frustrated trying to adjust to the variations in flight. Even with the same consistent quality of a shuttlecock, the behavior can be different depending on the local conditions. Experienced players know only too well that each badminton court behaves differentlty in terms of the flight of the shuttlecock. The attempt of this paper is to quantify those differences[1]. Any parameter that affects the density of air would affect the flight of the shuttlecock. This is because of the direct relationship between air density and air resistance. In this article, the effects of air temperature, humidity, and geographical altitude are considered and their effect on the shuttlecock's flight are analyzed.

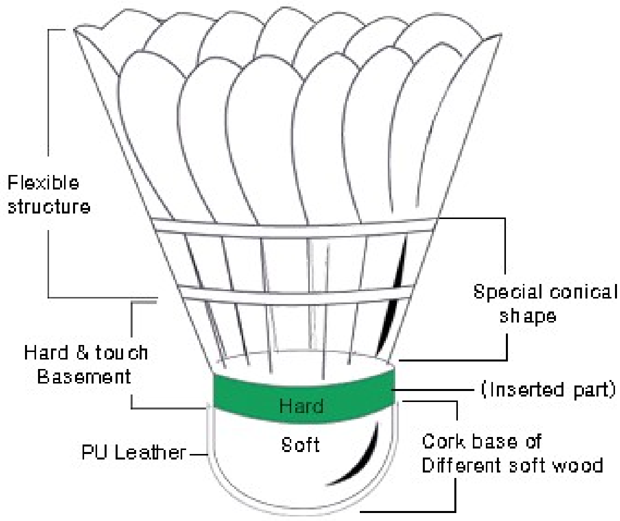

Figure 1: Illustration of parts of a feathered shuttlecock

The flight of a shuttlecock is actually a complex phenomenon. One of the most difficult things to model and analyze is the deformation that a shuttlecock undergoes at the point of impact and during the initial part of its travel. Some players are also able to impart spin to the shuttlecock, which increases the air resistance. Further, a shuttlecock is made up of many materials, which affects the turbulent behavior of air near the surface of the shuttlecock. In this analysis, all these effects are ignored, and with some justification. The deformation takes place for only a short period of time, and for the most part, the shuttlecock can be approximated to be a rigid object. Spin on the shuttlecock is somewhat of a specialized shot, and this can be ignored for the basic shots being analyzed. Some shuttlecocks have small amount of innate spin because of the arrangement of feathers, but since this is consistent it can be modeled by a concomitant increase in air resistance. The shuttlecock is modeled as a rigid object with a single specific air resistance that is a macro characteristic of the entire object.

1 Some badminton courts have a drift caused by tiny drafts of wind. Such effects are not analysed in this paper.

3 Theoretical Analysis

When a shuttlecock is in flight, it is subject to two forces – air resistance and gravity. The force due to gravity is straightforward:

$$F_g = m\,g \tag{1}$$

where m is the mass of the object, and g is the gravitational acceleration. This force always acts downwards.

The force due to air resistance is dependent on the density of air and the speed of the object. There is general consensus and experimental evidence [2], [3] that the air resistance is proportional to the square of the velocity of the shuttlecock. The force is given by

$$F_{air} = ½\,\rho\,A\,C\,v^2 \tag{2}$$

where ρ is the density of air, A is the effective area, C is the coefficient of air drag, and v is the speed of the object. Note that the direction of force here is directly opposite to the velocity of the shuttlecock.

At first glance, it appears that several parameters of the shuttlecock need to be measured – m, A and C – to analyze the effects of the forces. While the mass is easy to measure, it is not straightforward to measure A and C. Luckily there is an easy way out. A new parameter called the air resistance factor, q, is defined that is easy to measure and is sufficient for analysis.

When the shuttlecock is dropped from a height, it initially accelerates. As the speed increases, the force due to air resistance increases, and at a point it balances out the gravitational force. The speed at and after this point is called the terminal velocity. If dropped from a stationary position, it would take about 30 feet for the shuttlecock to reach within 1% of its terminal velocity (shorter if it is given an initial downward velocity), and can be relatively easily measured. Earlier studies [2] put typical terminal velocities at around 7 meters/sec at sea level and standard conditions. The actual number would be different for each shuttlecock, but it is reasonable to use this number for analysis.

Analyzing the forces at terminal velocity, the forces on the shuttle need to balance out. Hence we have

$$½\,\rho\,A\,C\,v_{terminal}^2 = m\,g \tag{3}$$

Let us define q, given by

$$q = A\,C\,/\,2m = \mathrm{g}\,/\,(\rho\,v_{terminal}^2) \tag{4}$$

The parameter q, defined as the air resistance factor, which is purely a characteristic of a shuttlecock. As is clear from its definition, the value of q does not depend on any external factor. If q is measured in different places, with different air density, or even different gravity, the measured value would be the same. For a typical shuttlecock, which has a terminal velocity of 7 meters/sec, at air density of 1.2 kg/m^3 (sea level, standard conditions), the value of q from equation 4 comes out to be 0.17. When measured at a place with lower air density, the terminal velocity would corresponsingly increase, yielding the exact same value for q.

To analyze the motion of the shuttlecock, the acceleration due to all the external forces need to be considered. The acceleration due to gravity is g in a vertically downward direction. The acceleration (more aptly deceleration) due to air resistance is opposite to the direction of the velocity, given by

$$a_{air} = -F_{air}/m = -½\,\rho\,A\,C\,v^2/m = -q\,\rho\,v^2 \tag{5}$$

The negative sign indicates that it is a deceleration. Note that this depends only on q, ρ and v. We have already seen how q can be measured fairly easily. ρ, the density of air, is a parameter of the system, g is a known global constant, and v is variable to be analyzed.

So far, only the magnitude of acceleration has been described. In general, the direction of air resistance is not the same as the direction of gravity. Using a Cartesian co-ordinate system that is in a vertical plane in which the shuttlecock travels in a badminton court as depicted in Figure 2,

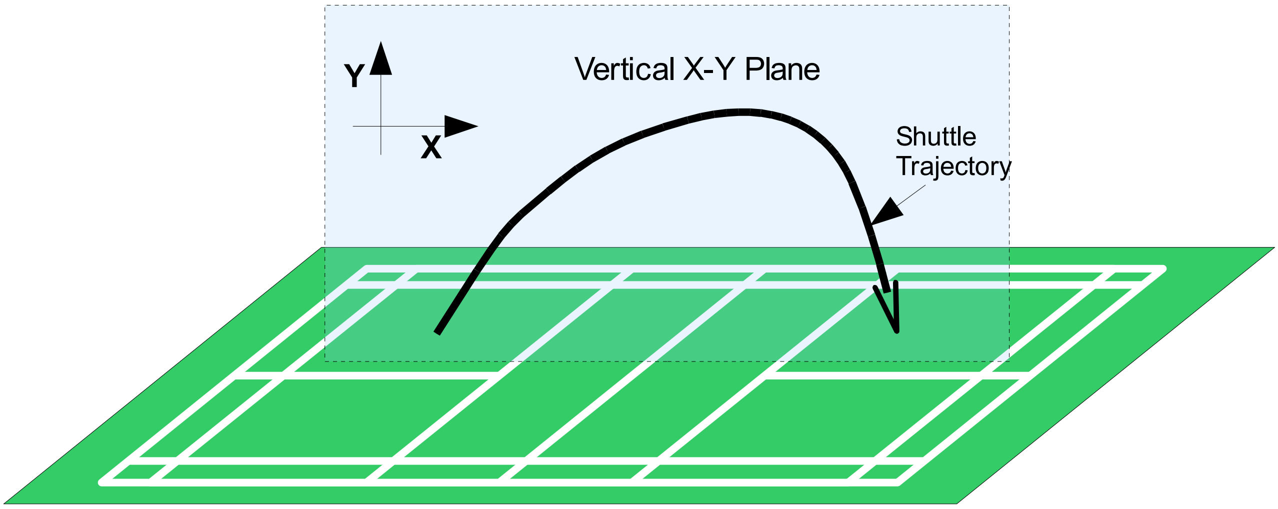

Figure 2: Vertical X-Y plane in which the shuttlecock travels

the total acceleration due to all forces is decomposed into a horizontal and vertical component, denoted by a_x and a_y. Similarly, the velocity is decomposed into v_x and v_y. The position of the shuttlecock in the vertical plane is denoted by x and y. With this we have

$$a_x = -q\,\rho\,v_x\,\text{sqrt}(v_x^2 + v_y^2)$$
$$a_y = -g - q\,\rho\,v_y\,\text{sqrt}(v_x^2 + v_y^2) \quad (6)$$

Since, by definition

$$a_x = dv_x/dt$$
$$a_y = dv_y/dt$$
$$v_x = dx/dt \quad (7)$$
$$v_y = dy/dt$$

Equation 6 can be re-written to solve v_x and v_y as

$$\frac{dv_x}{dt} = -q\cdot\rho\cdot v_x\cdot\sqrt{v_x^2+v_y^2}$$
$$\frac{dv_y}{dt} = -g - q\cdot\rho\cdot v_y\cdot\sqrt{v_x^2+v_y^2} \quad (8)$$

This system of differential equations can be solved to get the velocity given the initial conditions $v_x(0)$ and $v_y(0)$, air density ρ and the air resistance factor q. From the velocity, it is easy to calculate the trajectory of the shuttlecock, which is $\{x(t), y(t)\}$, given the initial conditions $x(0)$ and $y(0)$.[2]

Now, we look at all the parameters whose values are needed to get a numerical solution for the trajectory

1. $x(0)$, $y(0)$, $v_x(0)$ and $v_y(0)$: These are the initial position and velocity of the shuttlecock, whose values of are determined by the type of shot played.
2. q: The value of q depends only on the nature of the shuttlecock. For a typical shuttlecock the value of q, as already discussed after equation 4, comes out to be 0.17.
3. g: This is the gravitational acceleration, and is equal to 9.8 m/s^2. In theory this value does change with altitude. The change factor is $(R_0/R_1)^2$, where R_0 is the radius of the earth, and R_1 is distance of the place from the center of the earth (i.e. R_0 + altitude). For a place like Bangalore, which has an altitude of a little less than a kilometer, the change in gravity comes out to be -0.03%. This is quite insignificant, and can be ignored.
4. ρ: This is the density of air and is dependent on local conditions. The following section will describe the effects of various local conditions on ρ.

The factors that affect air density are [4]:

2 It is possible to solve Equation set 8 analytically. It is left as an exercise to the reader (which is a clever way of saying I don't know how to do it; it probably has some cosh, sinh or tanh). For purposes of this paper, a numerical solution is used.

1. Temperature
2. Humidity
3. Pressure, which is largely determined by the altitude of the location

3.1 Calculation of Air Density

Humidity (presence of water molecules in air) reduces the density of air. The molecular weight of water, which is 18, is lower than that of air, which is 28.97. Water vapor is present in air by displacing an equal volume of dry air. The density of humid air would be a weighted average of dry air and water vapor. The weights depend on the fraction of volume occupied by of each of the gases, which in turn is determined by the partial pressures of each of the gases.

In practice, the humidity of air is not described as an absolute partial pressure, but in relative terms. This relative humidity is defined as a ratio of the absolute partial pressure to the saturation vapor pressure at the given temperature and pressure. Since the saturation vapor pressure increases with temperature, the same relative humidity values (e.g. 40%) would mean different partial pressures at different temperatures. From [4], we have the partial pressure of water vapor at relative humidity, *r*, as

$$P_{vapor} = r \times 6.1078 \times 10^{\frac{7.5T - 2048.625}{T - 35.85}} \tag{9}$$

The total pressure is the sum of the partial pressures of water vapor and dry air

$$P = P_{vapor} + P_{dryair} \tag{10}$$

The density of dry air at a given temperature *T* and pressure *p* is given by

$$\rho_{dryair} = \frac{P \cdot M_{dryair}}{RT} \tag{11}$$

Where R is the universal gas constant (8.31447 J/(mol·K)), M_{dryair} is the molar mass of dry air, which turns out to be 0.0289644 kg/mol.

With a little bit of algebra, and application of gas laws, the density of humid air can be expressed as scaled version of the density of dry air at the same temperature and pressure:

$$\frac{\rho}{\rho_{dryair}} = \frac{P_{dryair}}{P} + \frac{P_{vapor}}{P} \cdot \frac{M_{vapor}}{M_{dryair}} \tag{12}$$

Where M_{dryair} and M_{vapor} are the molar masses of dry air and vapor respectively – we already know these values to be 28.9644 gm/mole and 18.0153 gm/mole respectively. Plugging in these values, and using (5) we can re-write this as

$$\rho = \rho_{dryair} \times \left(1 - \frac{0.37802 \cdot P_{vapor}}{P}\right) \tag{13}$$

The effect of altitude is a reduction in air pressure. This is governed by the expression([4], [5])

$$P = P_0 \cdot \left(1 - \frac{L \cdot h}{T_0}\right)^{\frac{g \cdot M}{R \cdot L}} \tag{14}$$

where

- sea level standard atmospheric pressure P_0 = 101325 Pa
- sea level standard temperature T_0 = 288.15 K
- Earth-surface gravitational acceleration *g* = 9.80665 m/s2.
- temperature lapse rate *L* = 0.0065 K/m
- universal gas constant *R* = 8.31447 J/(mol·K)
- molar mass of dry air *M* = 28.9644 gm/mol = 0.0289644 kg/mol

3.2 Summary of analysis

With this we (finally!) have all the 4 ingredients needed to compute the trajectory of a shuttlecock:

1. $x(0)$, $y(0)$, $v_x(0)$ and $v_y(0)$: the initial conditions, which depend on how the shuttlecock is struck.
2. q: the air resistance factor, which has a value of 0.17 SI units.
3. g: the gravitational acceleration, whose value is 9.8 m/s^2
4. ρ: the density of air for which we went through an elaborate rigmarole to determine it based on temperature, relative humidity and altitude using equations 9 through 14.

4 Results

The shuttlecock trajectory was computed for 3 different strokes – long serve, overhand clear, underhand clear and short serve. Two conditions were considered:

1. Air conditioned badminton court as sea level
   - Temperature = 22 deg C, Relative Humidity = 20%
2. Non air conditioned badminton court in a club in the city of Bangalore
   - Temperature = 29 deg C, Relative Humidity = 65%

4.1 Cautionary note on shuttlecock speeds

An assumption has been made that the shuttlecock is a rigid object, and that its air drag characteristics do not change with speed. This approximation is valid at lower speeds. However, at high speeds the shuttlecock gets deformed significantly. The feathers of the shuttlecock get bent inwards towards the axis of the shuttlecock, thus reducing the area presented to the air. This effectively reduces the air drag. In the following sections, the "initial" speed of the shuttlecock is presented for several types of strokes. These speeds do not account for deformation. The net result is that the initial speed is overestimated. The degree of overestimation is higher for high speed shots. However, this does not seriously undermine the findings in this paper. We shall see that the shuttlecock rapidly loses speed – for a powerful shot, it does down to 30% of its initial speed in just 0.2 seconds – and for a large part of the trajectory the speed is low enough for deformation to be ignored. The reader is cautioned not to take the initial speeds at face value. In reality, it would be somewhat lower. The analysis of deformation is beyond the scope of this paper.

4.2 Variation in Air density

The air density was calculated for each case using equations 9 through 14. Further, it was calculated for other cases to isolate the effect each of the local parameters. These are tabulated below:

No	Temperature	Relative Humidity	Altitude (feet)	Air Density	% reduction
1	22º C	20.00%	0	1.1942	0.00%
2	22º C	65.00%	0	1.1575	3.07%
3	29º C	20.00%	0	1.1653	2.42%
4	22º C	20.00%	3021[(3)]	1.0672	10.64%
5	29º C	65.00%	3021	1.0335	13.46%

Table 1: Variation of air density due to temperature, humidity and altitude

As expected, air density reduces with increase in temperature, humidity and altitude. The interesting thing to note is that altitude is the most significant contributor to reduction in air density.

4.3 Variation in trajectory of the shuttlecock

The variation in air density affects the manner in which the shuttlecock travels. A few commonly played shots are analyzed – a long serve, overhand clear, underhand clear and smash. At this point it would be useful for the reader to understand the dimensions of a badminton court. A badminton court with the dimensions marked is depicted in figure 3.

3 3021 feet is the mean altitude of the city of Bangalore, where I happen to reside.

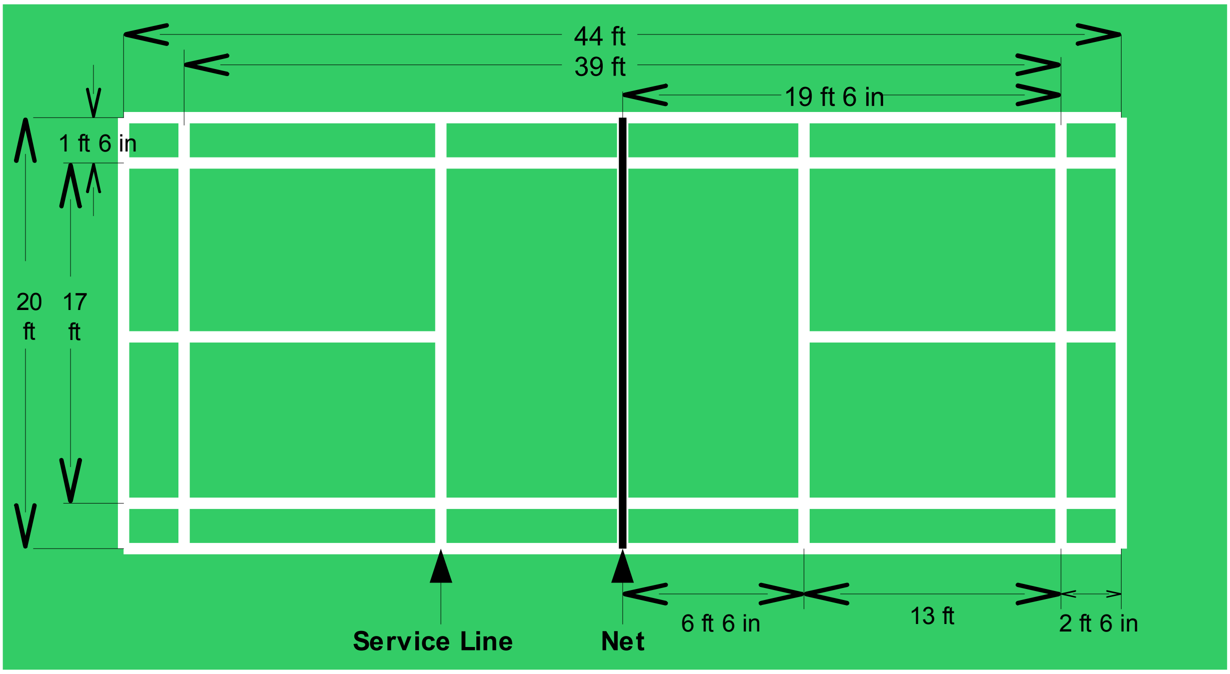

Figure 3: Dimensions of a badminton court

These shots are analyzed with the following points of view:

1. The variation in flight for different cases for the same shot played in each case.
2. Difference in shot played in each case in order to achieve the same (or similar) trajectory of the shuttlecock.

4.3.1 Long serve

A long serve is played from close to the service line on one court to the back end of the opposite court. In a singles game, a good long serve could have a length of up to 30 feet from the point of impact to the point where it lands on the ground. In doubles, it could up to 27 feet long. The serve could achieve a height of over 25 feet, more in singles than doubles. An average serve is considered with the following characteristics:

	Case-1	**Case-2**
Local conditions		
Temperature (C)	22.0	29.0
Rel. Humidity	20%	65%
Altitude (feet)	0	3021
Shuttlecock Characteristic		
Air Resistance factor (q)	0.170	0.170
Shot Characteristic		
Shot speed (kph)	100.0	100.0
Shot angle (deg)	60.0	60.0
Distance from Net (feet)	6.0	6.0
Height from ground (feet)	3.0	3.0

Note: The shot speeds are overestimated because shuttlecock deformation is not taken into account. (See section 4.1).

The trajectory for a typical long serve is depicted in figure 4.

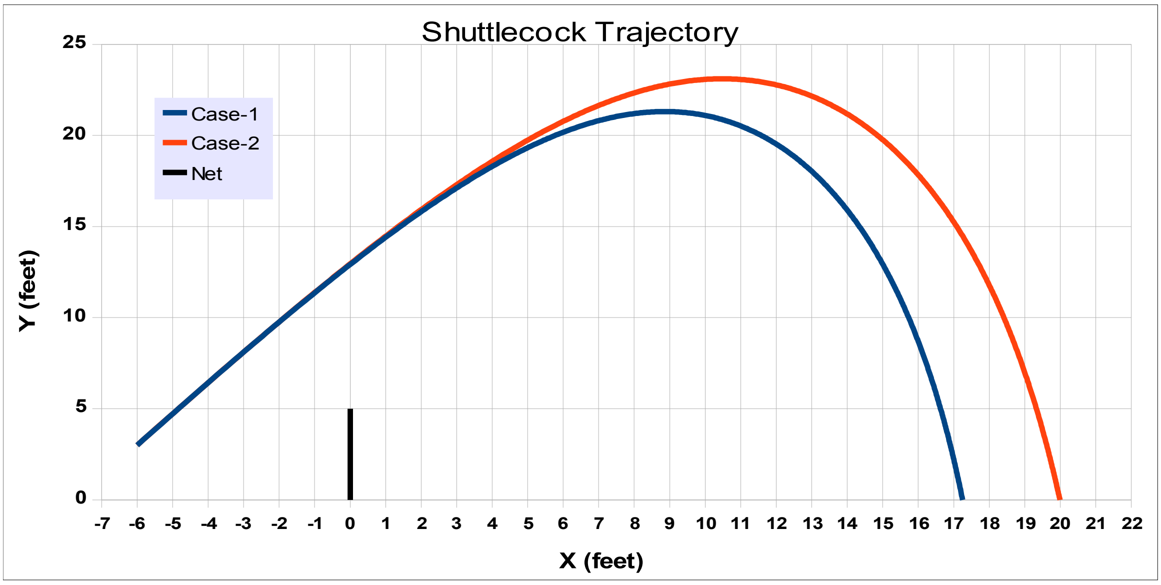

Figure 4: Trajectory of shuttlecock for a long serve

For the same shot, the speed of the shuttlecock is plotted across time in figure .

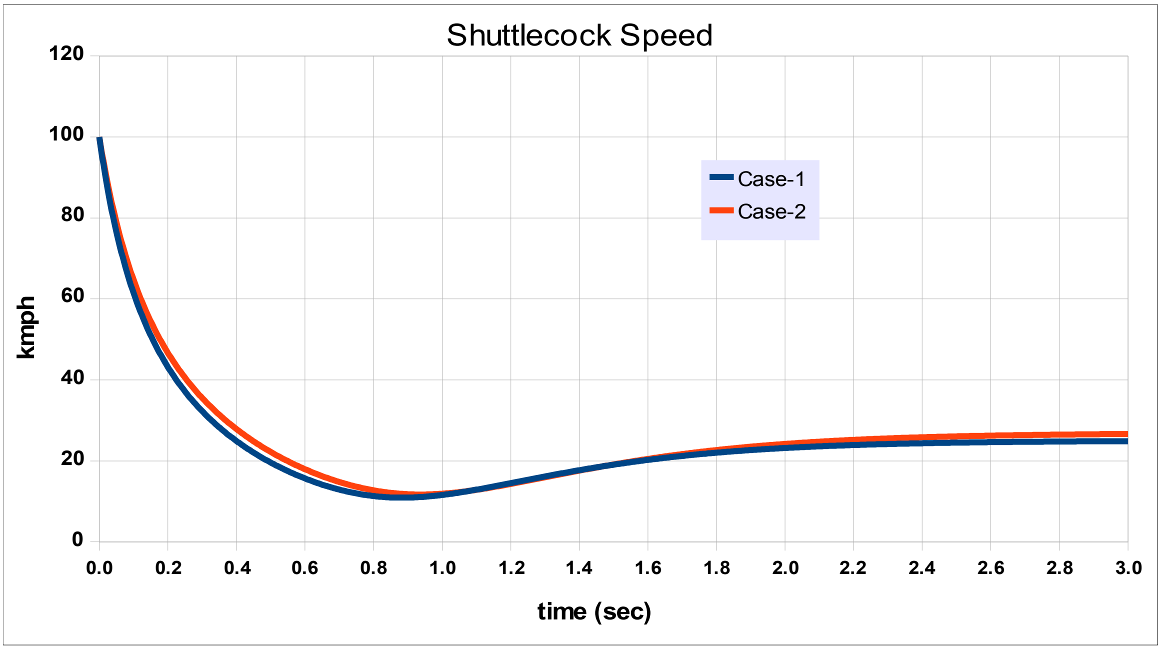

Figure 5: Shuttlecock speed across time for the long serve

The length and height of the shot achieved, and the time taken for the shuttlecock to reach the ground are summarized below:

Result	**Case-1**	**Case-2**
Shot Length (feet)	23.24	25.98
Shot Height (feet)	21.31	23.11
Time to hit ground (sec)	2.26	2.36

From the graphs, it is evident that in case 2, the shot travels longer by about 2 feet 9 inches. This a 11.8% increase in the length of the shot, which is quite significant.

Also note from figure 5 that the speed of the shuttlecock drops rapidly, and within 0.2 seconds, it down below 50% of its initial speed. The shuttlecock is the slowest at the top of its flight, and it then slowly accelerates and saturates towards its terminal velocity, which is a little above 25 kph

Now, for the same stroke, we attempt to match the trajectories as closely as possible, by varying the speed and angle of the shot for one of the cases. This will give us an idea about the extra effort needed in a place with higher air density. The speed and angle of the shot in case 1 have been adjusted to yield a trajectory as closely matched to case 2 as possible (It is theoretically impossible to get a perfect match). This is depicted in the figure below:

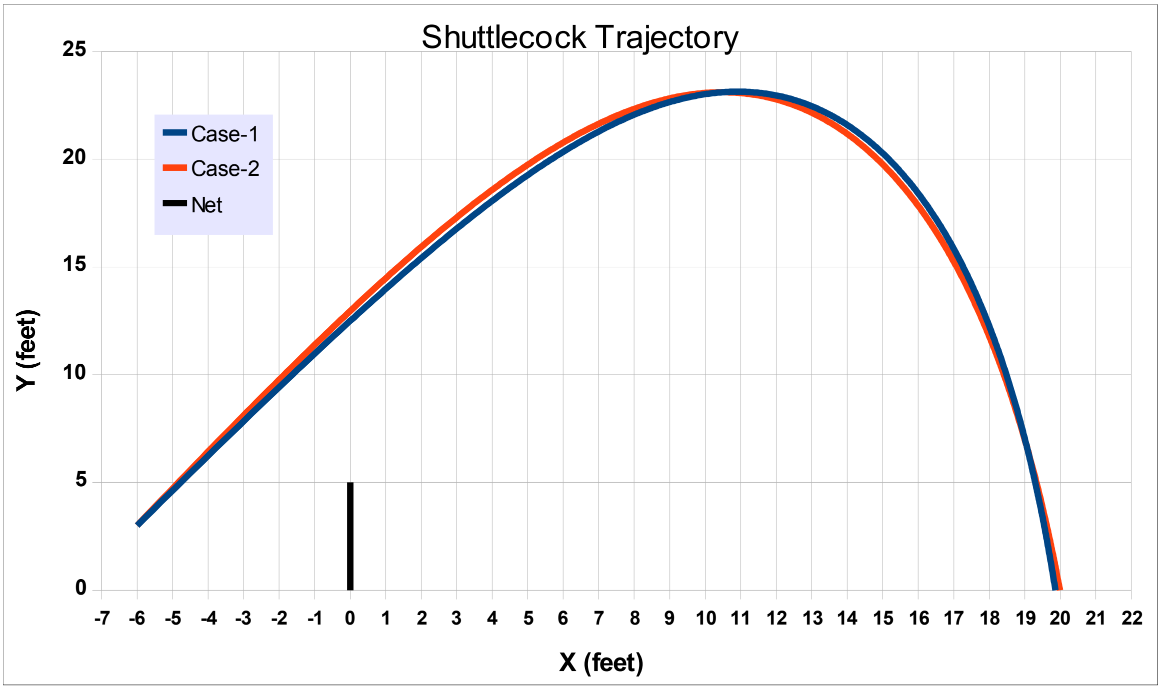

Figure 6: Closely matched trajectories for the long serve

The shot characteristics needed to achieve this match is given in the table below:

Shot Characteristic	**Case-1**	**Case-2**
Shot speed (kph)	119.5	100.0
Shot angle (deg)	58.5	60.0
Distance from Net (feet)	6.0	6.0
Height from ground (feet)	3.0	3.0

For case 1, it takes 19.5% greater initial shuttlecock speed to achieve a stroke similar to case 1. This represents a 42.8% increase in kinetic energy imparted to the shuttlecock (since kinetic energy is proportional to the square of the speed). There's no doubt that the game would be significantly more tiring for Case 1.

In this case, it also turns out that the shuttlecock takes about the same time to reach the ground (2.36 sec). So the reaction time for the opponent is unchanged.

Note: The shot speeds are probably overestimated because shuttlecock deformation is not taken into account. (See section 4.1).

4.3.2 Overhand clear

This is an overhead shot played from the back of the court high and deep into the back of the opposite court. This shot could travel over 40 feet in length and over 25 feet in height.

The shot characteristics are tabulated below. As before, we consider the case where the shot is played exactly the same way in the two conditions:

	Case-1	**Case-2**
Local conditions		
Temperature (C)	22.0	29.0
Rel. Humidity	20%	65%
Altitude (feet)	0	3021
Shuttlecock Characteristic		
Air Resistance factor (q)	0.170	0.170
Shot Characteristic		
Shot speed (kph)	175	175.0
Shot angle (deg)	50	50.0
Distance from Net (feet)	18.0	18.0
Height from ground (feet)	8.0	8.0

Note: The shot speeds are probably overestimated because shuttlecock deformation is not taken into account. (See section 4.1).

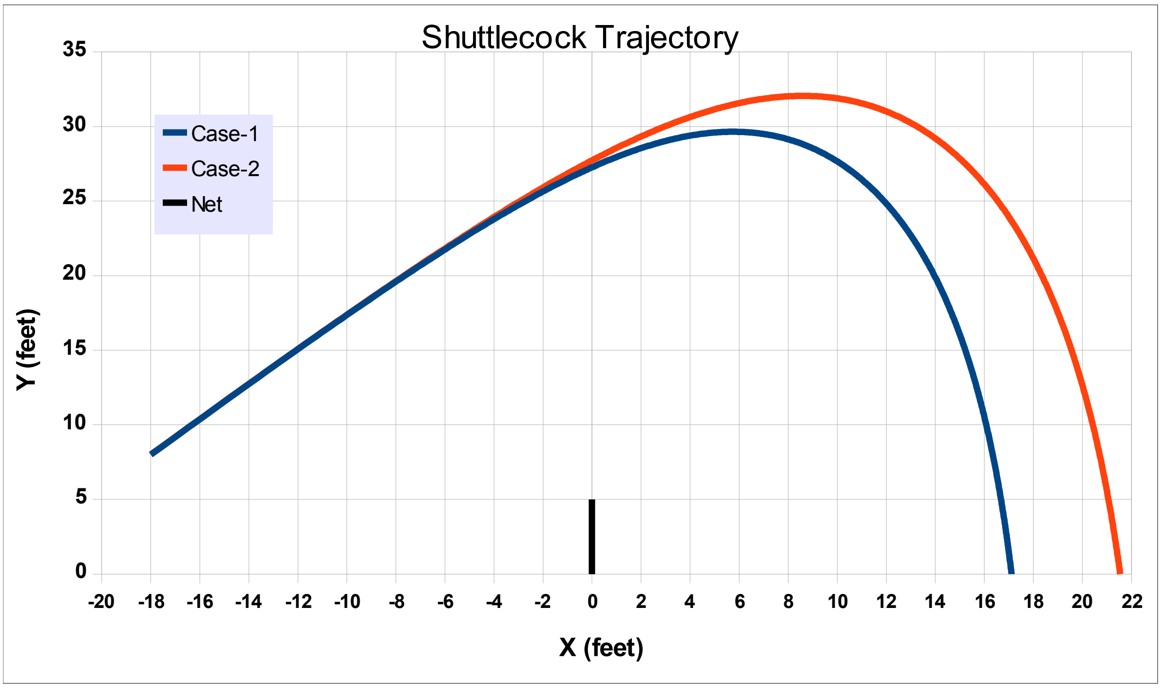

Figure 7: Trajectory of shuttlecock for a overhand clear

The trajectory of the shuttlecock for the overhand clear is plotted in figure 7.

The speed along time is depicted in figure 8:

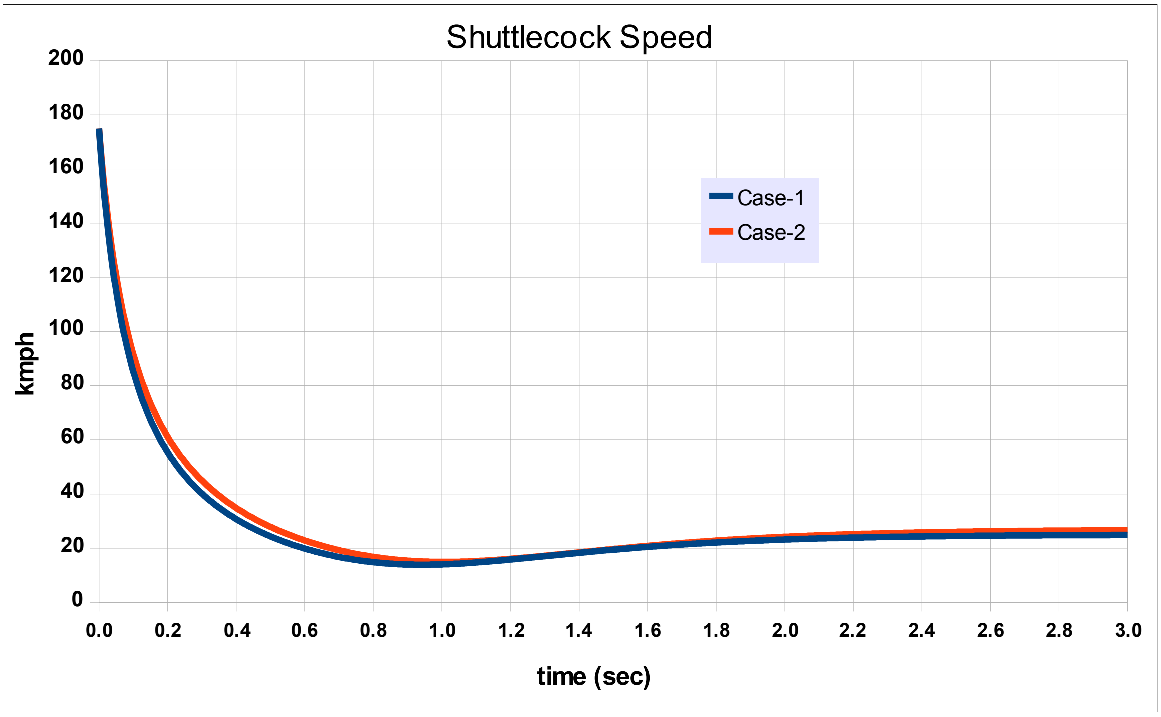

Figure 8: Shuttlecock speed across time for an overhand clear

The length, height and duration of the shot are tabulated below:

Result	**Case-1**	**Case-2**
Shot Length (feet)	33.76	37.91
Shot Height (feet)	29.64	32.05
Time to hit ground (sec)	2.1	2.18

The difference in length is more than 4 feet, which is similar to the long serve in terms of percentage.

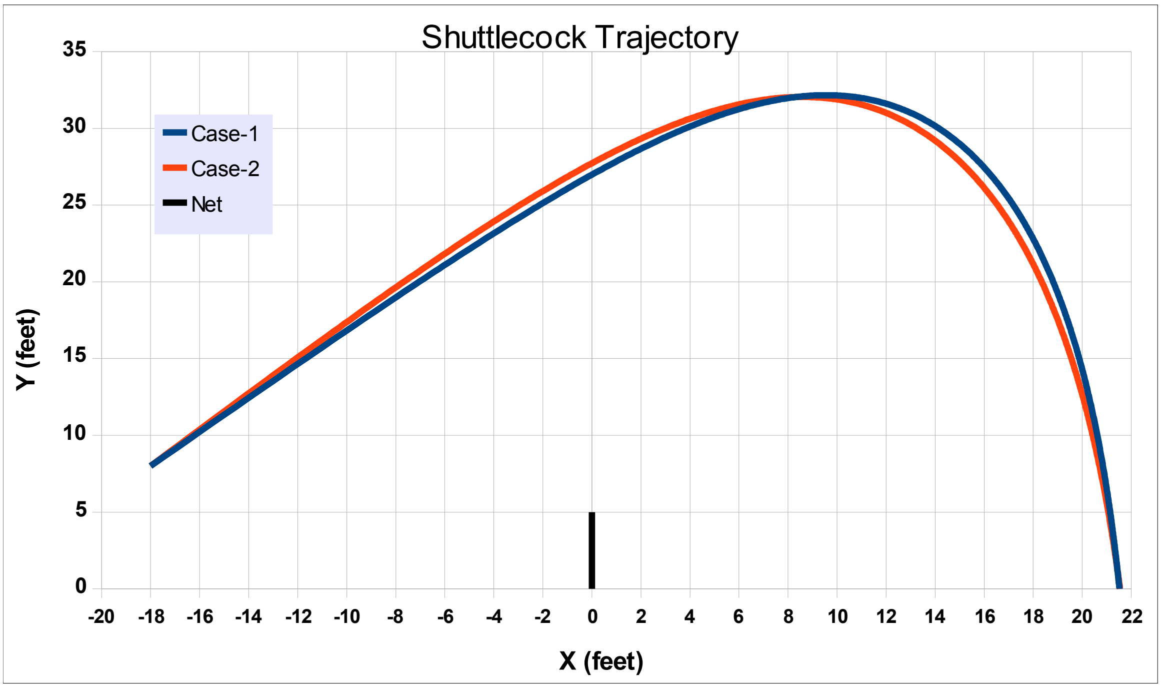

Figure 9: Closely matched trajectories for the overhand clear

Now, we again look at what extra effort it takes to achieve a similar shot in case 1 compared to case 2. The trajectory is depicted in figure 9.

To achieve this matching trajectory, the shot characteristics are:

Shot Characteristic	**Case-1**	**Case-2**
Shot speed (kph)	235.0	175.0
Shot angle (deg)	48.1	50.0
Distance from Net (feet)	18.0	18.0
Height from ground (feet)	8.0	8.0

Note: The shot speeds are probably overestimated because shuttlecock deformation is not taken into account. (See section 4.1).

The speeds are too high to make a meaningful analysis on effort required since shuttlecock deformation is bound to take place, and the actual effort needed would be lower than what the speeds project.

4.3.3 Underhand clear

An underhand clear is played from near the net high into the far end of the opposing court. Its trajectory is not too different from a long serve. The results are also similar. The results are presented below:

	Case-1	**Case-2**
Local conditions		
Temperature (C)	22.0	29.0
Rel. Humidity	20%	65%
Altitude (feet)	0	3021
Shuttlecock Characteristic		
Air Resistance factor (q)	0.170	0.170
Shot Characteristic		
Shot speed (kph)	85.0	85.0
Shot angle (deg)	60.0	60.0
Distance from Net (feet)	2.5	2.5
Height from ground (feet)	2.0	2.0

Flight trajectory is plotted below:

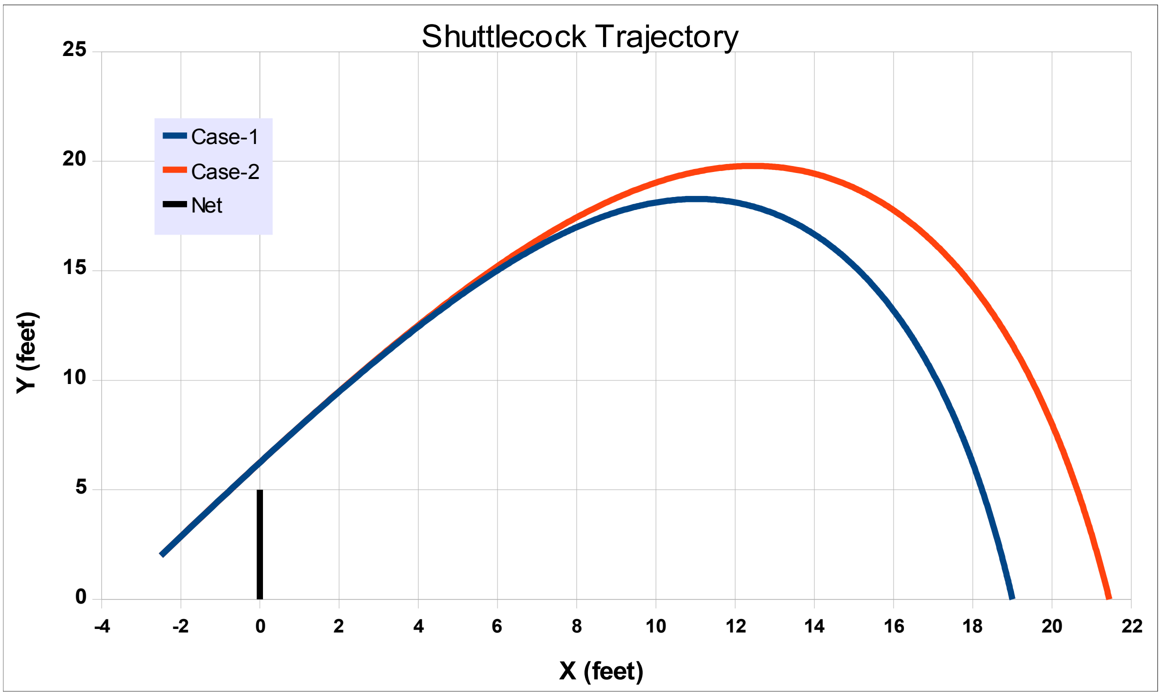

Figure 10: Shuttlecock trajectory for an underhand clear shot

Flight speed is plotted along time below:

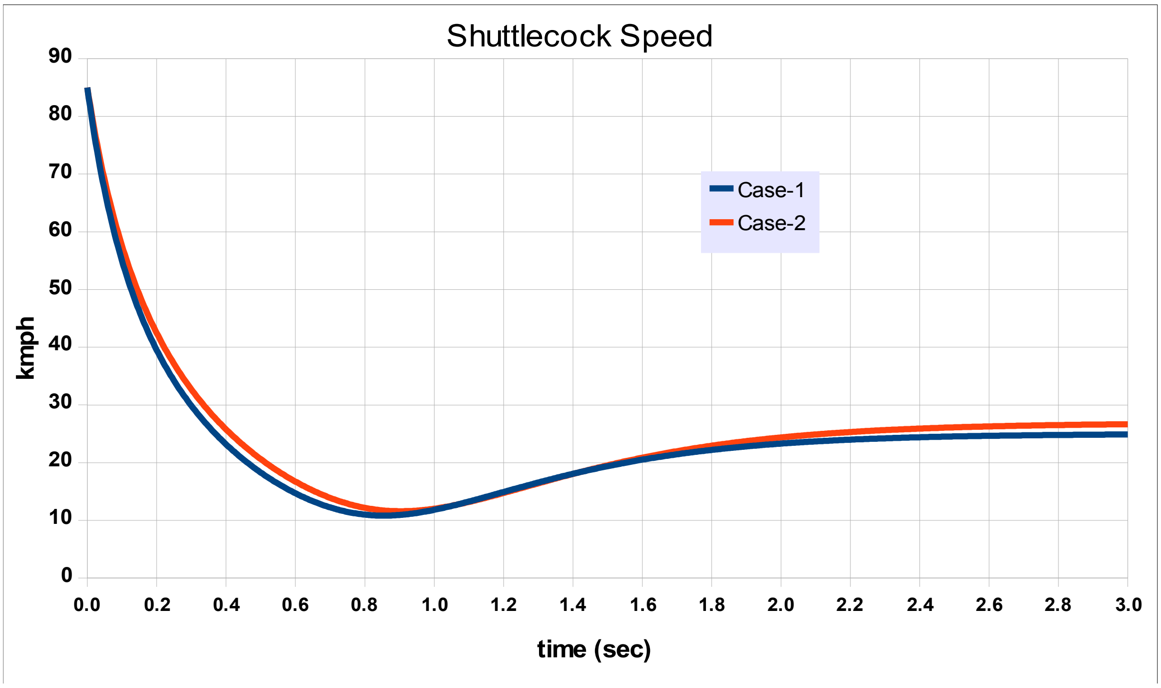

Figure 11: Shuttlecock speed across time for an underhand clear

The length, height and duration are given below

Result	**Case-1**	**Case-2**
Shot Length (feet)	21.49	23.95
Shot Height (feet)	18.28	19.79
Time to hit ground (sec)	2.1	2.18

4.3.4 Short serve

A short serve is a played from near the service line, traveling just over the net and just beyond the service line of the opposite court. The length of the serve is around 15 feet. The shot characteristics are tabulated below:

	Case-1	Case-2
Local conditions		
Temperature (C)	22.0	29.0
Rel. Humidity	20%	65%
Altitude (feet)	0	3021
Shuttlecock Characteristic		
Air Resistance factor (q)	0.170	0.170
Shot Characteristic		
Shot speed (kph)	32.0	32.0
Shot angle (deg)	32.0	32.0
Distance from Net (feet)	6.0	6.0
Height from ground (feet)	3.0	3.0

The trajectory of the shuttlecock is plotted below

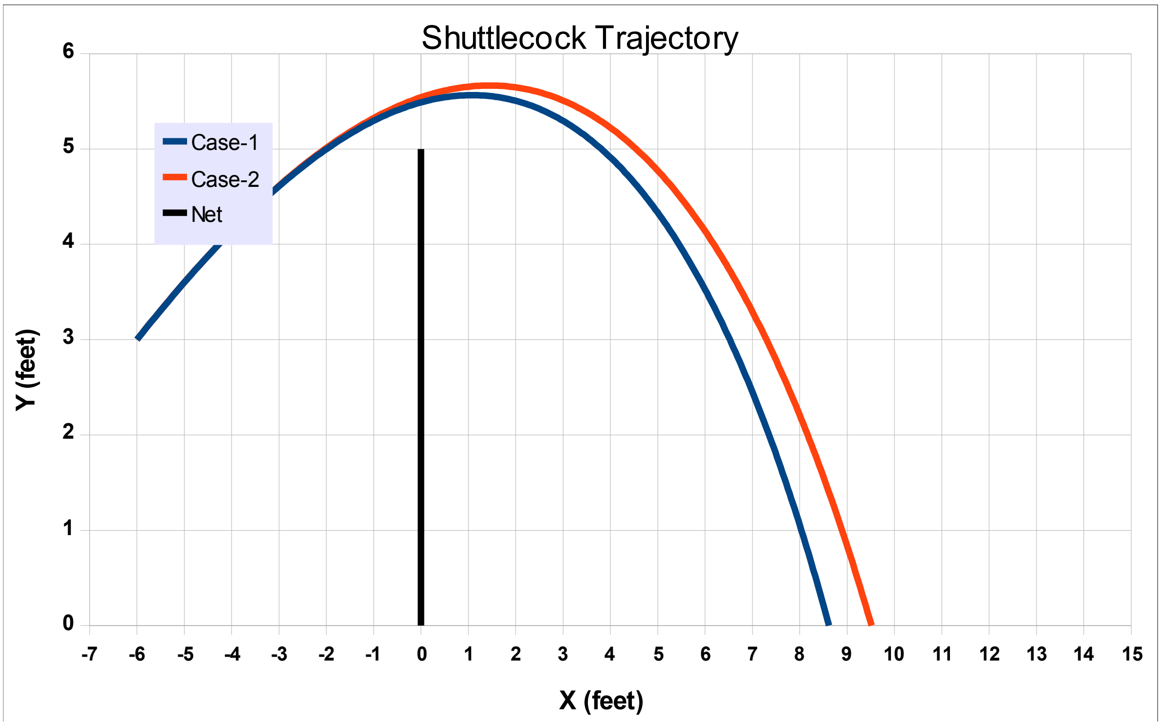

Figure 12: Shuttlecock trajectory for a short serve

The speed is plotted against time below

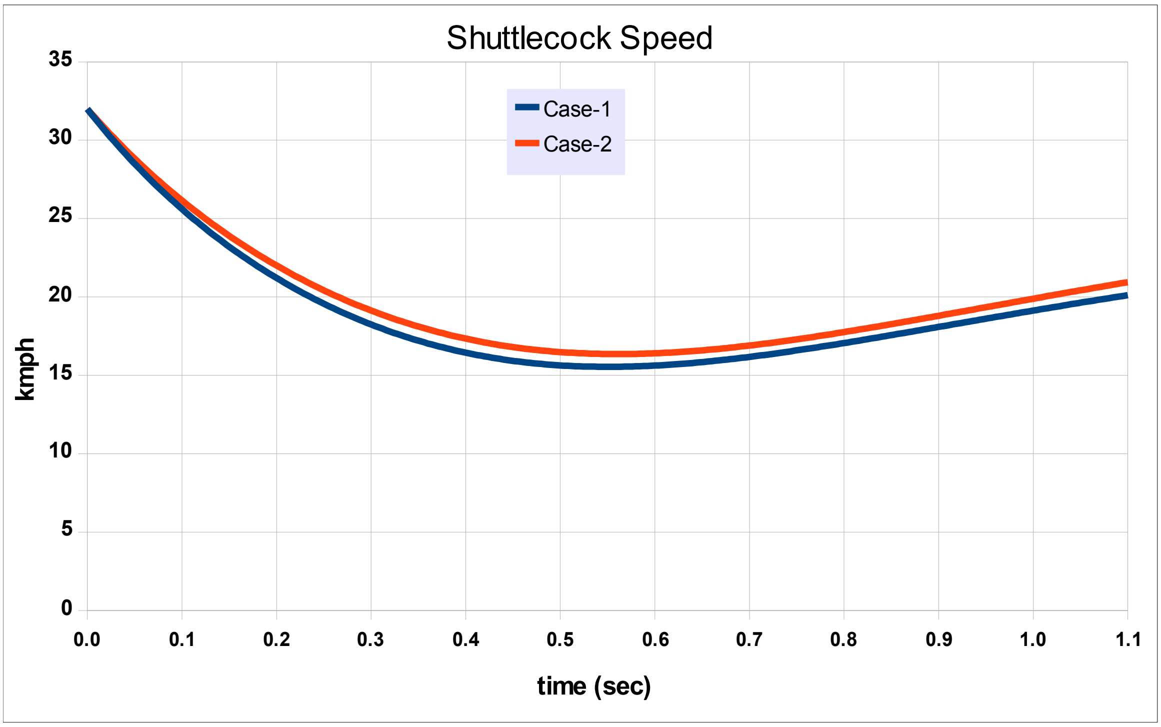

Figure 13: Shuttlecock speed across time for a short serve

The length, height and duration are

Result	Case-1	Case-2
Shot Length (feet)	14.6	15.51
Shot Height (feet)	5.57	5.67
Time to hit ground (sec)	1.02	1.03

Interestingly, even for a short serve there is a difference of over 10 inches, which is about 6 percent.

5 Conclusion

The effect of temperature, humidity and altitude on the flight of a feathered shuttlecock was analyzed. All three of these conditions manifest themselves as a change in air density, which alters the nature of flight. The most pronounced effect was that of altitude. Temperature and humidity have a smaller effect. Results for a particular set of conditions were presented for several common badminton strokes. It is known that local conditions affect the flight of the shuttle, In this paper, these changes were quantifies. Results show that there is a significant change in the trajectory of the shuttlecock under different conditions – in some cases a 3 feet increase in the length of the shot.

6 References

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