Legendre's Conjecture → Andrica's Conjecture?

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Lemma 0.1. *For all $n > 0$, let p_n denote the n-th prime number, then $\sqrt{p_{n+1}} - \sqrt{p_n} < 1$.*

Proof. The following inequalities are equivalent for $n > 0$,

$$\begin{aligned} n^2 + 1 &< n^2 + 2n + 1 \\ n^2 - 2n + 1 &< n^2 + 1 \\ (n-1)^2 &< n^2 + 1 \\ 2(n-1) &< 2\sqrt{n^2+1} \\ n^2 + 2n &< n^2 + 2\sqrt{n^2+1} + 2 \\ \sqrt{n^2+2n} &< 1 + \sqrt{n^2+1} \\ \sqrt{n^2+2n} - \sqrt{n^2+1} &< 1 \end{aligned}$$

By Legendre's Conjecture, we know there is a prime p such that for all $n > 0$, $n^2 < p < (n+1)^2$. In other words, $n^2 + 1 \leq p \leq (n+1)^2 - 1 = n^2 + 2n$ and our proof is complete.