



Fig 1

$\begin{pmatrix} 5 \\ 3 \end{pmatrix}$

A cylindrical vessel lies on its side. The ends of the vessel are circular and are depicted in Fig 1 as a circle, radius r , centred on the origin. The vessel is filled half-full of liquid, of density ρ , such that the level of the liquid lies on the x -axis - at $y = 0$.

Our task is to find the force exerted on the end-plate of the vessel by the fluid.

Divide the bottom half of the end-plate into a number of strips. Let there be n strips in all with each strip of thickness $\delta y = r/n$.

The width of the strip is given by L . Since the strip is very thin, we can assume, as a reasonable approximation, that the pressure at a depth y is the same as the pressure at a depth $y + \delta y$.

Knowing the area of this strip and the pressure acting on it, we can calculate

the force acting on this strip. If we do the same for all the strips involved, and add up the forces on all of the strips, we can get a reasonable approximation to the total force acting on the end-plate.

Consider an elemental strip of the circle at a depth y below the level of the fluid. This thickness of the strip is denoted by δy and its half-width by $L/2$. Using Pythagoras' theorem, $L/2 = \sqrt{r^2 - y^2}$.

Let δA be the area of the strip, $\delta A = 2 * (L/2) * \delta y = L\delta y$.

The pressure in a liquid at a depth h below its surface is given by $p = \rho gh$. Therefore, at the depth y , the pressure on the elemental strip, from the liquid, is given by $p = \rho gy$

The force on the strip is

$$\begin{aligned}\delta F &= p\delta A \\ \delta F &= \rho gy * L\delta y \\ \delta F &= 2\rho gy\sqrt{r^2 - y^2}\delta y\end{aligned}$$

We can find the total force on the end-plate by summing the forces on all the strips. This becomes

$$\begin{aligned}F &= \sum \delta F \\ F &= \sum 2\rho gy\sqrt{r^2 - y^2}\delta y\end{aligned}$$

In the limit, as n , the number of strips, $\rightarrow \infty$ and $\delta y \rightarrow 0$ we get the integral summation

$$\begin{aligned}F &= \int 2\rho gy\sqrt{r^2 - y^2} dy \\ F &= 2\rho g \int_0^r y\sqrt{r^2 - y^2} dy \\ F &= -\frac{2}{3}\rho g \left[(r^2 - y^2)^{\frac{3}{2}} \right]_0^r \\ F &= -\frac{2}{3}\rho g \left[(0)^{\frac{3}{2}} - (r^2)^{\frac{3}{2}} \right] \\ F &= \frac{2}{3}\rho gr^3\end{aligned}$$

Example

$$\rho = 42 \text{ lb}/ft^3$$

$$g = 32 \text{ ft}/s^2$$

$$r = 1.5 \text{ ft}$$

$$F = \frac{2}{3} * 42 * 32 * 1.5^3$$

$$F = 3,024 \text{ lbf}$$

$$F = 94.5 \text{ lbs}$$