Proof for v^2/r
(No Calculus Required)

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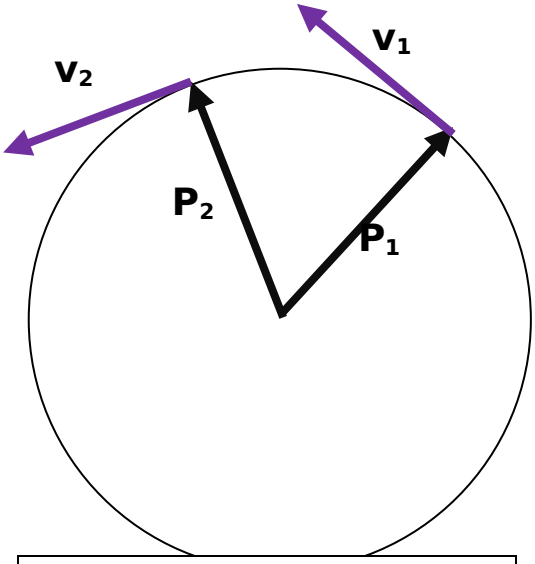
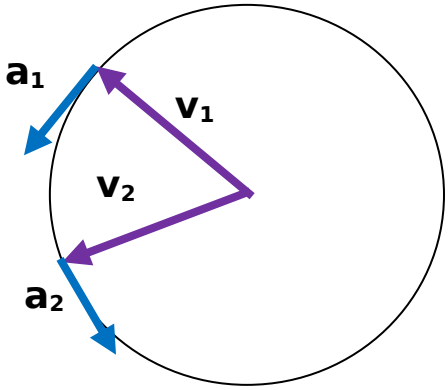
Vector Proof for $a_c = v^2/r$

Imagine two circular orbits, one to map the effect of change in velocity on the position and one to map the effect of acceleration on velocity.



Effect of velocity on position (Circle P)



Effect of acceleration on velocity (Circle V)

Where the vectors $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{a_1}$, $\mathbf{a_2}$ are tangential to the respective circles. Note that each of the corresponding velocity vectors is the **same**.

Circumference of Circle P: $2\pi r$
Period (Time taken to travel around the circle):
T = $2\pi r/v$

Doing the same thing with circle V,
T = $2\pi v/a$

Solve for a

$$2\pi r/v = 2\pi v/a$$
$$r = \frac{v^2}{a}$$
$$a = \frac{v^2}{r}$$

And you're done!