



Solution:

Assume the beam to have a length L and dia d . Let the crane rope hook at a length l from the pivot. At some instant, let the angle made by the beam be α . The major forces on the beam are 1) It's own weight which is supposed to act at the centroid if you are considering a uniform beam. 2) Tension in the crane rope 3) Load supported by the surface. Fig 1 can be converted to the equivalent Fig 2 by shifting the forces from the centre line to a parallel line of action (along the pivot edge). Due to this shift, moments due to $W \sin \alpha$ and $T \sin \alpha$ must be taken.

Taking equilibrium of moments about A,

$$W \cos \alpha \times \frac{L}{2} - W \sin \alpha \times \frac{d}{2} - T \cos \alpha \times l + T \sin \alpha \times \frac{d}{2} = 0$$

Which gives the tension in the rope as:

$$T = \frac{W \times (L \cos \alpha - d \sin \alpha)}{2l \cos \alpha - d \sin \alpha}$$

Again, at equilibrium, the total vertical force must be zero. This gives:

$$R + T = W.$$

Substituting the value of T which was obtained above, we get the reaction of the ground

$$R = \frac{W \cos \alpha (2l - L)}{2l \cos \alpha - d \sin \alpha}$$

Special case: When crane rope is supporting at the end of the beam (i.e.) $l = L$:

1) when $\alpha = 0$ degrees, $R = W/2$ which is as expected since the beam is uniform and simply supported at both ends.

2) when $\alpha = 90$ degrees, $R = 0$! This is true since the beam is supported throughout its lift by the rope of the crane (The rope will always be in tension (i.e.) It does not slack!). So at 90 degrees, for equilibrium, the entire load has to be balanced by the rope. When the rope is released (or slacked), the entire load of the beam is transferred to the ground.

If there is no rope to support the load ($T=0$), the beam automatically falls back to its stable (vertical) position when the angle of inclination (α) is greater than $\tan^{-1}(\frac{L}{d})$. If the length of the beam (L) is much greater than its diameter (d), L/d tends to infinity and $\tan^{-1}(\frac{L}{d})$ tends to 90 degrees. Eg: if length of the beam is 100m and its dia is 2m, $L/d = 50$ and the angle after which beam automatically falls and becomes vertical = 88.8 degrees.