There are 4 principles all our work is based on.

I. IVT (**Intermediate value theorem**) If a function f is continuous on an interval, then the values it takes also form an interval.

Hence if there exist points a,b in the domain interval with $f(a) < K < f(b)$, then f(x) must equal K at some c between a and b.

E.g. the continuous function f(x) = x^3 + x + 1 must equal zero for some x between -1 and 0, since f(-1) = -1, and f(0) = 1.

II. EVT (**Extreme value theorem**, also called Max Min Value or MMV)

If a function is continuous on a closed bounded interval, then its values also form a closed bounded interval, in particular there is a (finite) smallest and a (finite) largest value.

I.e. if f is continuous on [a,b] then there exist c,d in [a,b] such that for every x in [a,b], $f(c) \leq f(x) \leq f(d)$.

E.g. since the right circular cone of height R+x, inscribed in a sphere of radius R, has volume (1/3)(R^2-x^2)(R+x), which is continuous for $0 \leq x \leq R$, one of these cones has largest volume.

III. (Rolle's Theorem) If f is continuous on [a,b] and differentiable on (a,b), and if f(a) = f(b), then f achieves a maximum (or minimum) at a point c strictly between a and b where f'(c) = 0, i.e. at a "critical point" for f. E.g. f(x) = x^3-x on [0,1] has a min. at x = sqrt(1/3).

Cor: If f has two derivatives on [a,b] and f' takes the same value twice, e.g. if there are two critical points, and if f'' is zero only a finite number of times, then f has a flex somewhere on (a,b) where f''=0. E.g. f(x) = x^3 – 3x on [-1,1] has a flex at x = 0.

Cor: If f is continuous on [a,b] but has no critical points in the interval (a,b), then f cannot take the same value twice in [a,b], hence cannot change direction, i.e. f is strictly monotone on [a,b]. Using I, f thus has an inverse function defined on [f(a),f(b)], (or [f(b),f(a)]). E.g. f(x)= x^3 +x +1 has an inverse defined on the whole real line.

Cor: If f '' exists but is never zero on [a,b], then f never changes concavity on [a,b], i.e. f is either concave up or concave down on all of [a,b]. E.g. f(x) = x^2 is concave up everywhere.

IV. MVT **(Mean value theorem)** If f is continuous on [a,b] and differentiable on (a,b), there is a point c with $a < c < b$, and f'(c)(b-a) = f(b)-f(a).

Cor: If f'=g' on [a,b] then f-g is constant on [a,b]. (Since f'(x)(x-a) = g'(x)(x-a) for all $a \leq x \leq b$, then f(x)-f(a) = g(x)-g(a), for all $a \leq x \leq b$, so f(x)-g(x) = f(a)-g(a), for all x.)

Cor: Since for continuous f, d/dx ($\int_a^x f$) = f(x), then $\int_a^b f$ = G(b)-G(a), for any G with G' = f.

E.g. since d/dx(x^3/3) = x^2, thus $\int_0^1 x\text{^}2$ = 1^3/3 – 0^3/3 = 1/3.