2.18

Let $A = \sum_{n=0}^{\infty} C_{2n}$ and $B = \sum_{n=0}^{\infty} C_{2n+1}$

where $n = 0, 1, 2, \ldots$

and let p be the polynomial

$p(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$

Then $A^2 = \left[\sum_{n=0}^{\infty} C_{2n}\right]^2$ and $B^2 = \left[\sum_{n=0}^{\infty} C_{2n+1}\right]^2$

Hence,

$A^2 = (C_0 + C_2)^2 = C_0^2 + 2C_0C_2 + C_2^2$

$B^2 = (C_1 + C_3)^2 = C_1^2 + 2C_1C_3 + C_3^2$

So, $A^2 - B^2 = (C_0^2 + 2C_0C_2 + C_2^2) - (C_1^2 + 2C_1C_3 + C_3^2)$

$= C_0^2 - C_1^2 + C_2^2 - C_3^2 + 2C_0C_3 - 2C_1C_3$

and,

$p(1) = C_0 + C_1 + C_2 + C_3$

$p(-1) = C_0 - C_1 + C_2 - C_3$

So, $p(1)p(-1) = (C_0 + C_1 + C_2 + C_3)(C_0 - C_1 + C_2 - C_3)$

$= C_0^2 - C_0C_1 + C_0C_2 - C_0C_3 + C_0C_1 - C_1^2 + C_1C_2 - C_1C_3$
$+ C_0C_2 - C_1C_2 + C_2^2 - C_2C_3 + C_0C_3 - C_1C_3 + C_2C_3 - C_3^2$

$= C_0^2 - C_1^2 + C_2^2 - C_3^2 + 2C_0C_3 - 2C_1C_3$

Hence $A^2 - B^2 = p(1)p(-1)$ as required.