RESONANT FREQUENCIES IN ACOUSTICAL CAVITIES

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INTRODUCTION:

Resonance occurs when a given system is driven externally to vibrate at greater amplitude at a certain preferred frequency. This frequency is known as the **resonant frequency**, which corresponds to the natural frequency of vibration of the object or system. For an acoustical cavity, the resonant frequencies will depend upon the dimensions of the cavity, its geometry, and the speed of sound at the temperature of the air.

For this experiment, you will determine the resonant frequencies of a rectangular acoustical cavity both theoretically and experimentally.

THEORY:

Rectangular cavity

Imagine a closed cavity of length *l*. The lowest frequency standing wave mode in this cavity will be such that the maximum dimension of the cavity is one half of the wavelength. When this is the case, the end walls of the cavity will see zero particle velocity (also known as a **velocity node**) and maximum pressure variations, as shown in Fig. 1.

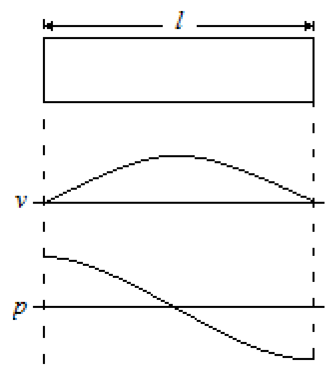

Figure 1 Velocity and pressure diagrams for the lowest frequency standing wave mode shown for a cavity of length *l*

This frequency *f* is given by:

$$f = \frac{v}{2l} \tag{1}$$

where *v* is the velocity of sound waves in the gas and *l* is the length of the cavity.

Now, if we extend this idea to a rectangular cavity, we see that there will be fundamental modes when a standing wave exists of the two dimensions, just like the standing wave from the one dimensional cavity. If we go one dimension further to the three dimensional box, the volume can be thought of as being divided into thin rectangular cells. Fundamental modes in this volume occur when each rectangular plane has a velocity node.

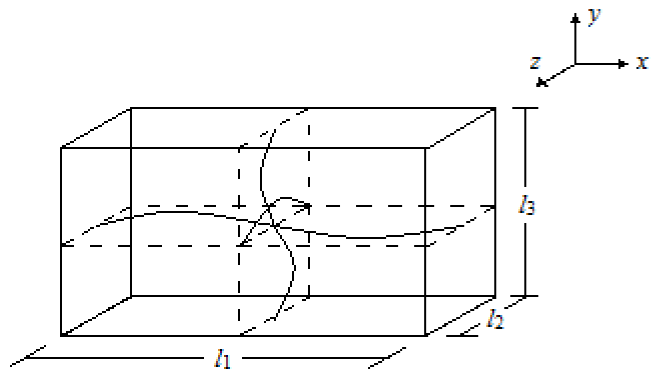

Figure 2 Three-dimensional acoustical cavity (like the one dealt with in \ this experiment) with dimensions l_1, l_2, and l_3 corresponding to the *x*, *y*, and *z* axes respectively.

If the dimensions of this cavity are l_1, l_2, and l_3 (as shown in Fig. 2) and the integers *p*, *q*, and *r* are respectively associated with the dimensions, then the frequency for any set of integers in this three-dimensional cavity is given by:

$$f^2 = \frac{v^2}{4}\left[\left(\frac{p}{l_1}\right)^2 + \left(\frac{q}{l_2}\right)^2 + \left(\frac{r}{l_3}\right)^2\right] \qquad (2)$$

Eq. 2 (developed by Rayleigh) is derived from the idea of the **wave equation**, a second-order partial differential equation that all waves must follow. This wave equation is:

$$\frac{1}{v^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} \equiv \nabla^2\phi \qquad (3)$$

where the function ϕ (*x*, *y*, *z*, *t*) is the displacement for air molecules for sound.

If we assume that ϕ varies sinusoidally, for any of the eigentones it can be expressed as $\sin 2\pi f/v$, then we may rewrite Eq. 3 in the following form:

$$\nabla^2\phi + k^2\phi = 0 \qquad (4)$$

Here, *k* is given by $k = 2\pi f/v$. For the pipe cavity, there is a boundary condition that needs to be satisfied. This condition, that $d\phi/dN = 0$, where *N* is the normal to the surface, must be satisfied over the surface of the entire cavity. We see that the following form for ϕ satisfies the boundary condition:

$$\phi = \cos(p\pi x/l_1)\cos(q\pi y/l_2)\cos(r\pi z/l_3) \qquad (5)$$

This solution satisfies the form of the wave equation given in Eq. 4.

$$k = \pi \left[{p^2}/{l_1{}^2} + {q^2}/{l_2{}^2} + {r^2}/{l_3{}^2} \right]^{1/2} \tag{6}$$

From our earlier definition of the value *k*, we see that we can solve that for the frequency to get $f = vk/2\pi$. If we substitute Eq. 6 into this expression for the resonant frequency, we arrive at the following result:

$$f = {vk}/{2\pi} = {v}/{2} \left[{p^2}/{l_1{}^2} + {q^2}/{l_2{}^2} + {r^2}/{l_3{}^2} \right]^{1/2} \tag{7}$$

We can see that by simply squaring Eq. 7 we will arrive at Eq. 2. The derivation of the equation used in this experiment is based off of the theory of the wave equation, and is clearly shown in the steps above.

When we are using this equation to calculate for the resonant frequencies, any of two of the integers *p*, *q*, and *r* may be zero, essentially meaning that at least one of the integers must be nonzero.

At room temperature (about 21°C), the speed of sound in air is approximately 343 m/s. Below, we provide a table with sample frequency calculations for an acoustical cavity that is 1.0 meter (l_1) by 1.5 meters (l_2) by 2.0 meters (l_3) at room temperature. You will be doing similar calculations corresponding to the dimensions of your box.

p	***q***	***r***	***f* (Hz)**
0	0	1	85.75
0	1	0	114.33
1	0	0	171.50
0	1	1	142.92
1	0	1	191.74
1	1	0	206.12
1	1	1	223.24
0	0	2	171.50
0	1	2	206.12
1	0	2	242.54

Table 1 Sample calculations for acoustical cavity 1.0 m x 1.5 m x 2.0 m at room temperature. While we stopped our calculations after finding ten distinct frequencies, it is easy to imagine calculating more with varying combinations of *p*, *q*, and *r*.

Speed of sound

These frequencies are easy to calculate if the velocity of sound is known. The velocity v of the propagation of sinusoidal waves through any fluid (for example, air) is given by:

$$v = \sqrt{\frac{B}{\rho}} \tag{8}$$

where B is the **bulk modulus** of the fluid and ρ is the density of the fluid.

The bulk modulus indicates a substance's resistance to uniform compression. Specifically, it is equal to the change in pressure Δp needed to change a volume V by ΔV:

$$B \equiv -V\, {}^{dp}\!/_{dV} \tag{9}$$

Air has an approximate adiabatic bulk modulus of 1.42×10^5 Pa. However, we can ignore this approximate value and continue with our derivation for the speed of sound. For an ideal gas compressed adiabatically, the following equality holds:

$$pV^{\gamma} = \text{constant} \tag{10}$$

where $\gamma \equiv C_p/C_v$, or the ratio of specific heats of the gas. (C_p is the specific heat at constant pressure and C_v the specific heat at constant volume.) So $B = \gamma p$. Thus:

$$v = \sqrt{{}^{\gamma p}\!/_{\rho}} = \sqrt{{}^{\gamma RT}\!/_{M}} \tag{11}$$

where T is the absolute temperature of the gas and M is the molecular weight. R is the gas constant. $\gamma = 1.667$ for monatomic gasses like argon and $\gamma = 1.400$ for diatomic gasses like oxygen.

Quality factor of the cavity

The cavity exhibits resonance, and like all resonant systems it has energy losses and a quality factor Q which can be measured. The quality factor is a measure of the sum of such energy losses. Energy losses occur due to the following factors:

1) Viscosity of air
2) Energy transfer to the walls of the cavity
3) Energy loss from the source to the receiver (the speaker to the microphone)

Q of an oscillator is defined as the ratio between the energy stored in the oscillator to the energy dissipated per radian, or:

$$Q = \frac{\text{energy stored in oscillator}}{\text{energy dissipated per radian}}$$

You can estimate the quality factor of your cavity by measuring the full width at half maximum (FWHM) of the Fourier transform of a resonant frequency. Note that the *y*-axis of the Fourier transform on the digital oscilloscope is given in decibels, so you will have to convert this to get the correct Q factor.[1] Another option is to quickly shut off the function generator at resonance, record the damped sound wave, and determine the quality factor from there.

For theoretical and experimental information about resonance in cylindrical and spherical cavities, see the references by Moloney and Russell, both from the American Association of Physics Teachers.

EQUIPMENT:

- Rectangular acoustical cavity*
- Speaker
- Function generator
- Digital oscilloscope
- Ruler or meter stick
- Amplifier
- Microphone
- Microphone stand or probe

* Your rectangular acoustical cavity should have two openings, one of which is a hole drilled into one of the sides to fix the speaker to. The other should be large enough to fit your microphone in.

PROCEDURE:

Check to see that the rectangular acoustical cavity has a hole or small opening in one of the sides. (If there is no opening to be found, either drill a hole in yourself or ask your lab technician to locate a cavity with a proper opening.

Measure each side of the acoustical cavity and record the data in your lab notebook. Remember to measure the dimensions of the *inside* of the cavity, not the outside.

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[1] The vertical axis of the FFT display on the oscilloscope is logarithmic, displayed in dBV (decibels relative to 1 volt RMS). 1 dBV $= 20\log(V_{\text{RMS}})$. Thus, a 1 V RMS sine wave (2.8 V from peak to peak) will read 0 dBV on the FFT display.

Attach your speaker to the cavity so that it covers the hole, and fix with tape or some other adhesive. Wire the speaker to the function generator so that when the generator is turned on, the frequency will play a sound at the specified frequency. Wire your microphone so that it is connected to the amplifier, which is in turn connected to a digital oscilloscope. A simple diagram of this setup is further illustrated in Fig. 3.

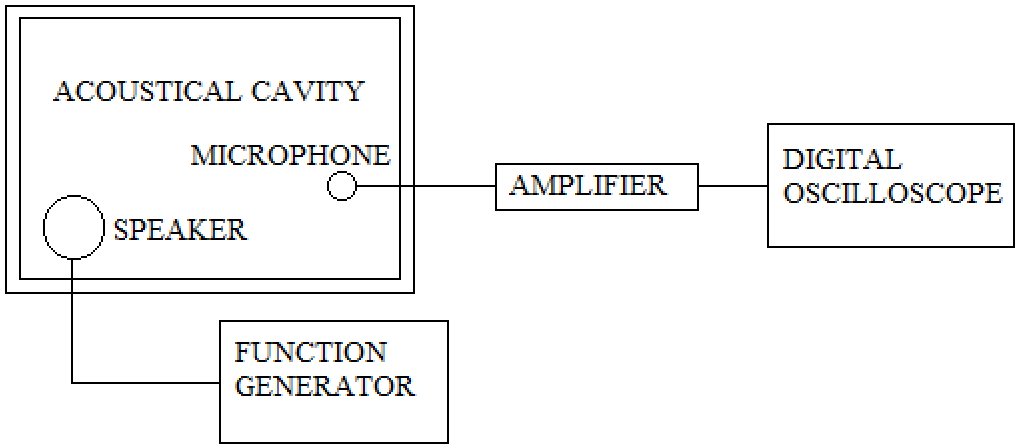

Figure 3 Basic diagram of the experimental setup.

First, determine the speed of sound in the conditions that you are running the experiment under. Turn on the function generator and the amplifier, and set the output frequency of the generator to be around 100 Hz. If your acoustical cavity is considerably large, set the starting frequency to be even lower, around 50 Hz. Make sure that you are able to hear the sound from the speaker before adjusting anything else.

From here, slowly increase the frequency of the signal emitted until resonance occurs in the cavity. If you listen carefully, there should be a frequency at which there is a slight decrease in the volume when you slightly increase the frequency. (Ignoring resonance, the sound will generally become louder as you increase the frequency.) This should be the lowest frequency of resonance. You can use this value to solve for the speed of sound in the material.

Now that you know the speed of sound, use Eq. 2 to find the theoretical frequencies for different values of p, q, and r, following the example from Table 1. To see if your theoretical calculations align with actual results, set the frequency of the function generator within the ballpark of the calculated theoretical frequencies, and check for resonance.

Once you have set the function generator on one of the resonant frequencies, you can insert your microphone into the acoustical cavity with some kind of probe and move it throughout the cavity. As you move it, notice where the signals on the digital oscilloscope reach their maximum or minimum. Record these positions in your notebook. Repeat this process for other resonant frequencies as well.

You can also detect see the resonant frequency in displayed in the digital oscilloscope through a Fourier transform. To do so, press the “Math menu” button on the oscilloscope and select the function to read “FFT.” As you play a frequency, you should see a sharp spike in your digital oscilloscope of that frequency. As you increase or decrease the frequency, this spike should move along with it. At the resonant frequency the height of spike should increase, since the box will also be vibrating at the driving frequency.

MORE QUESTIONS TO CONSIDER:

We gave you the theory and equation for a closed cavity, and the values for the frequencies of each mode were calculated (see example calculations in Table 1). Suppose that you were given the same box, except with one face removed. (This is an open-sided cavity.) What would be the new eigenfrequencies? Make a table similar to Table 1 listing the first 10 lowest modes and their eigenfrequencies. If you have time, you can also experimentally investigate this system to see if your calculations were correct.