We can work out the integral of the first term on the RHS using the same technique that was applied to the original, magnetostatic version of Ampère's law. If you refer back to that derivation, we transformed the x component of the integrand to the form

$$-\left(\mathbf{J}\left(\mathbf{r}'\right)\cdot\nabla_{r'}\right)\frac{x-x'}{\left|\mathbf{r}-\mathbf{r}'\right|^{3}} = \boxed{-\frac{x-x'}{\left|\mathbf{r}-\mathbf{r}'\right|^{3}}\nabla_{r'}\cdot\mathbf{J}\left(\mathbf{r}'\right)} + \boxed{\nabla_{r'}\cdot\left(\frac{x-x'}{\left|\mathbf{r}-\mathbf{r}'\right|^{3}}\mathbf{J}\left(\mathbf{r}'\right)\right)} \qquad (11)$$

where the derivatives are now with respect to the primed coordinates. The second term on the RHS can be converted to a surface integral using the divergence theorem and, if the currents are localized, works out to zero. In the magnetostatic case $\nabla_{r'}\cdot\mathbf{J}\left(\mathbf{r}'\right)=0$, but here that isn't true, as we saw above. However, if we return to three dimensions are look again at the integral, we get

$$\begin{aligned} -\int\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^{3}}\nabla_{r'}\cdot\mathbf{J}\left(\mathbf{r}'\right)d^{3}\mathbf{r}' &= \int\dot{\rho}\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^{3}}d^{3}\mathbf{r}' \qquad (12)\\ &= \frac{\partial}{\partial t}\int\rho\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^{3}}d^{3}\mathbf{r}' \qquad (13)\end{aligned}$$

However, the integral is essentially the definition of the electric field:

$$\mathbf{E}=\frac{1}{4\pi\epsilon_{0}}\int\rho\frac{\mathbf{r}-\mathbf{r}'}{\left|\mathbf{r}-\mathbf{r}'\right|^{3}}d^{3}\mathbf{r}' \qquad (14)$$

Combining this with 10, we get

$$\nabla\cdot\left(\nabla\times\mathbf{B}\right)=\mu_{0}\nabla\cdot\mathbf{J}+\mu_{0}\epsilon_{0}\nabla\cdot\dot{\mathbf{E}} \qquad (15)$$