Closed form expressions for RF MEMS switch actuation and release time

V. Kaajakari

Closed form expressions for RF MEMS switch actuation and release times are derived. The expressions account for the mechanical and electrical parameters and are valid for $V > 1.15\,V_P$ and $Q > 0.1$ where V is the actuation voltage, V_P is the pull-in voltage, and Q is the mechanical quality factor.

Introduction: RF MEMS switches offer substantially higher performance than conventional diode or transistor based RF switches [1, 2]. For many applications, the switching speed is a critical specification. The switches are based on mechanical movement between the open and closed states, and the actuation speed is limited by the mechanical response time. The actuation dynamics is nonlinear and numerical modelling has been used to characterise the switch dynamics [3]. This approach is accurate but does not offer design intuition.

In this Letter, we derive simple analytical expressions for the switching actuation and release times. The expressions are valid for typical switch operation regimes and allow evaluation of the switch performance and its sensitivity to process variations.

Switch dynamics: The RF MEMS switch shown in Fig. 1*a* is governed by the equation of motion

$$m\frac{\partial^2 x}{\partial t^2} + \gamma\frac{\partial x}{\partial t} + kx = \varepsilon\frac{A}{2(d-x)^2}V^2 \tag{1}$$

where m is the movable electrode mass, γ is the damping coefficient, k is the spring constant, x is the movable electrode displacement, ε is the permittivity of a free space, A is the electrode area, d is the open state electrode gap, and V is the actuation voltage. Equation (1) is nonlinear and does not have a closed form solution. The switch displacement as a function of time can be obtained using numerical integration as shown in Fig. 1*b*. We make the following observations.

1. The switch spends the majority of the time reaching the half-way point. The final half of the gap is closed rapidly. For most of the time, the gap is relatively large and the gap dependent damping can be taken as constant.
2. The numerical simulations closely resemble the switch behaviour shown in Figure 3 in [3]. This further justifies neglecting the gap dependency in the damping coefficient.
3. Owing to switch inertia, the switch moves initially slowly. Later, the damping limits the switch velocity.

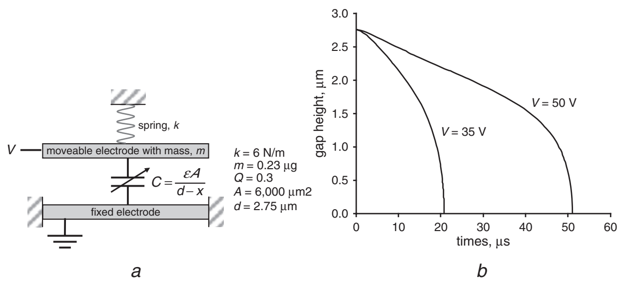

Fig. 1 *Operation of capacitive RF MEMS switch*

a Model for capacitive switch [3]
b Results of numerical integration of (1)

Actuation time: If the spring and damping forces can be ignored, we can obtain a closed form expression for the switching time. The work done by the capacitive actuator in moving from the open position to location x_a is

$$W = \int_0^{x_a} F\,\mathrm{d}x = \frac{1}{2}V^2\frac{\mathrm{d}C}{\mathrm{d}x} = \varepsilon\frac{A}{2(d-x_a)}\frac{x_a}{d}V^2 \tag{2}$$

If the damping coefficient and the spring constant are zero, this work is done against the switch mass, which will gain kinetic energy $\frac{1}{2}m\dot{x}^2 = W$. Solving for the switch velocity $\dot{x}$, we have

$$\dot{x} = \sqrt{\varepsilon\frac{A}{m(d-x_a)}\frac{x_a}{d}V^2} \tag{3}$$

From velocity $\dot{x} = (\mathrm{d}x/\mathrm{d}t)$, the time to travel a small distance $\mathrm{d}x$ is $\mathrm{d}t = \mathrm{d}x/\dot{\mathrm{x}}$. Using (3), the inertia limited switch actuation time is

$$t_m = \int_0^d \frac{\mathrm{d}x_a}{\dot{x}} = \frac{\pi}{2V}\sqrt{\frac{d^3}{\varepsilon A m}} = \sqrt{\frac{27}{32}}\frac{\pi}{\omega_0}\frac{V_P}{V} \tag{4}$$

where we have used the expression $V_P = \sqrt{(8/27)(kd^3/\varepsilon A)}$ for the pull-in voltage [1, 2].

Another limiting case for the switch closing time is obtained by ignoring the switch inertia. By setting $m = 0$ and solving for the velocity, (1) gives

$$\dot{x} = \frac{1}{\gamma}\left(\varepsilon\frac{A}{2(d-x)^2}V^2 - kx\right) \tag{5}$$

The damping limited switch actuation time is obtained from integrating $\mathrm{d}t = \mathrm{d}x/\dot{x}$, which does not have closed form solution. The spring force kx, however, is relatively small at small displacements and we can simplify the integrand with a Taylor series expansion to give

$$\begin{aligned} t_\gamma &= \int_0^d \frac{\mathrm{d}x}{\dot{x}} = \int_0^d \frac{2\gamma(d-x)^2}{A\varepsilon_0 V^2}\frac{1}{1 - 2kx(d-x)^2/A\varepsilon_0 V^2}\mathrm{d}x \\ &\simeq \int_0^d \frac{2\gamma(d-x)^2}{A\varepsilon_0 V^2}\left[1 + \frac{2kx(d-x)^2}{A\varepsilon_0 V^2} + \left(\frac{2kx(d-x)^2}{A\varepsilon_0 V^2}\right)^2\right]\mathrm{d}x \\ &= \frac{2\gamma d^3}{315}\frac{5d^6k^2 + 21d^3K\varepsilon_0 AV^2 + 105\varepsilon_0^2A^2V^4}{\varepsilon_0^3A^3V^6} \end{aligned} \tag{6}$$

Using the pull-in voltage, (6) can be written as

$$t_\gamma = \frac{V_P^2}{Q\omega_0 V^2}\left(2.25 + 1.52\frac{V_P^2}{V^2} + 1.22\frac{V_P^4}{V_4}\right) \tag{7}$$

A good general approximation for the switch actuation time is obtained by summing the inertial and damping limited switching times given by (4) and (7), respectively, to give

$$t_{\mathrm{close}} = t_m + t_\gamma \tag{8}$$

For the switch in Fig. 1, (8) gives switch actuation times of 25 and 50 µs for $V = 35$ V and V = 50 V, respectively, which agrees well with the 24 and 51 µs in [3].

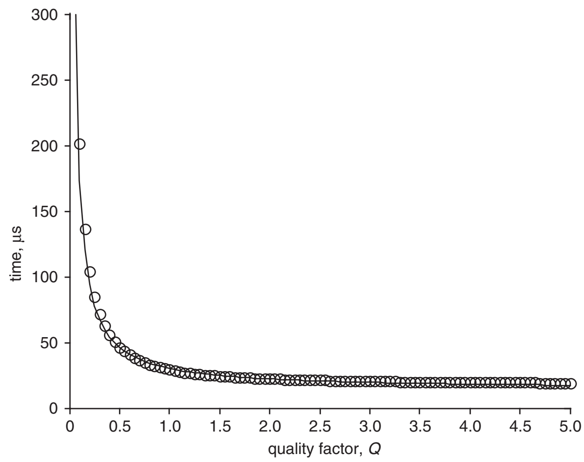

Fig. 2 *Simulated (circles) and approximate calculated (solid line) switch closing times for V = 1.2V*$_P$

Fig. 2 shows a comparison of the analytical switch closing time given by (8) and results from the direction numerical integration of (1). The maximum error for $V > 1.15\,V_P$ and $Q > 0.1$ is less than 20%. To reduce the contact bounce, the quality factor for the RF MEMS switches is usually $0.2 < Q < 5$ [1], which is covered by the approximate expression (8). For actuation voltages smaller than $V = 1.15\ V_P$, the switching time grows as the switch spends significant time reaching the pull-in point and the Taylor series expansion in (6) is no longer accurate.

Switch release time: The actuator opening time can be estimated in a similar fashion. Ignoring the damping, the switch release time can be estimated by the principle of energy conservation $(1/2)m\dot{x}^2 = (1/2)kd^2 - (1/2)kx^2$. Solving for the velocity gives

$$\dot{x} = -\sqrt{(kd^2 - kx^2)/m} \tag{9}$$

The opening time is obtained from integrating $\mathrm{d}t = \mathrm{d}x/\dot{x}$ from $x = d$ to $x = 0$ giving

$$t_m = \int_d^0 \frac{\mathrm{d}x}{\dot{x}} = \frac{\pi}{2\omega_0} \tag{10}$$

If damping is significant and inertial effects can be ignored, the switch opening time is obtained by solving

$$\gamma\frac{\partial x}{\partial t} + kx = 0 \tag{11}$$

which has the solution $x = de^{-(\gamma/k)\,t}$. The time for the switch to be 80% open is $t = \gamma/k\ \ln 5 \simeq 1.6\ (1/\omega_0 Q)$. Thus, depending on the quality factor, the switch release time is

$$t_{\mathrm{open}} = \begin{cases} \dfrac{\pi}{2\omega_0} & \text{for } Q > 1 \\ 1.6\dfrac{1}{\omega_0 Q} & \text{for } Q < 1 \end{cases} \tag{12}$$

Equation (12) is accurate to 20%.

Conclusion: Closed form expressions derived for RF MEMS switch actuation and release times depend only on the mechanical quality factor, the resonance frequency, the actuation voltage, and the pull-in voltage. The expression can be used to design the switch to meet the desired actuation and release times. The optimal operation regime is seen to be $0.5 < Q < 2$, where switching speed is not limited by damping but excessive ringing and switch bounce is avoided. For lower quality factors, the damping significantly increases the switching times.

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V. Kaajakari (*Louisiana Tech University, Institute for Micromanufacturing, 911 Hergot Drive, Ruston, LA 712 72, USA*)

E-mail: ville@latech.edu

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