The photoionisation cross section, starting from Fermi's golden rule in the dipole approximation is given by

$$\sigma = \left[\left(\frac{\xi_{eff}}{\xi_0} \right)^2 \frac{n_r}{\varepsilon} \right] \frac{4\pi^2}{3} \alpha_{fs} h\nu \sum_f \left| \langle \Psi_i \, | \overrightarrow{r} | \, \Psi_f \rangle \right|^2 \delta \left(E_f - E_i - h\nu \right), \quad (1)$$

where

$$\delta \left(E_f - E_i - h\nu \right) = \frac{1}{\pi} \frac{\hbar \Gamma_f}{\left(E_f - E_i - h\nu \right)^2 + \left(\hbar \Gamma_f \right)^2}. \quad (2)$$

The energy

$$\lambda_{nm} = \gamma^2 \left(2n + \beta + 1 \right) - \frac{\gamma^4}{2}$$

The wave of function of the electron confined in the ring is given by

$$\Psi_{nm} \left(r, \theta \right) = \frac{1}{\sqrt{2\pi}} \frac{\gamma}{R} \sqrt{\frac{\Gamma \left(n+1 \right)}{\Gamma \left(\beta + n + 1 \right) 2^{\beta}}} \left(\gamma\rho \right)^{\beta} e^{-\frac{1}{4}\gamma^2\rho^2} L_n^{\beta} \left(\frac{\gamma^2 \rho^2}{2} \right) e^{-im\theta}, \quad (3)$$

where

The Matrix element, considering two states with both $n = 0$ and $m = 0$ and $m = 1$ for the initial and final states, polarization in the x axis, respectively is

$$\begin{aligned} \langle \Psi_{00} \, | \overrightarrow{x} | \, \Psi_{01} \rangle = \frac{\gamma^2}{2\pi} R \sqrt{\frac{1}{\Gamma \left(\beta_i + 1 \right) 2^{\beta_i}}} \sqrt{\frac{1}{\Gamma \left(\beta_f + 1 \right) 2^{\beta_f}}} \int_0^{\infty} \int_0^{2\pi} \left(\gamma\rho \right)^{\left(\beta_i + \beta_f \right)} \\ e^{-\frac{1}{2}\gamma^2\rho^2} \cos\theta e^{-i\theta} \rho^2 d\rho d\theta \end{aligned} \quad (4)$$

where the energies are

$$\begin{aligned} \lambda_{00} &= \gamma^2 \left(\beta_i + 1 \right) - \tfrac{\gamma^4}{2} \\ \lambda_{01} &= \gamma^2 \left(\beta_f + 1 \right) - \tfrac{\gamma^4}{2}. \end{aligned} \quad (5)$$

and

$$\begin{aligned} \beta_i &= \gamma^2 / 2 \\ \beta_f &= \sqrt{1 + \gamma^4 / 4} \end{aligned} \quad (6)$$

and the constants

$$\begin{gathered} m_e = 0.067 m_0, m_0 \text{free electron mass} \\ \varepsilon = 13.1 \\ \mathit{\Gamma} = 1/2.0 s^{-1} \\ n_r = 3.2. \\ \xi_{eff} / \xi_0 = 1 \\ \alpha_{fs} = e^2 / \hbar c \end{gathered}$$

And the values of $\gamma = 1.0, 1.5$ e 2.0.