

0.1 D'Alembert's mechanics: beginners beware

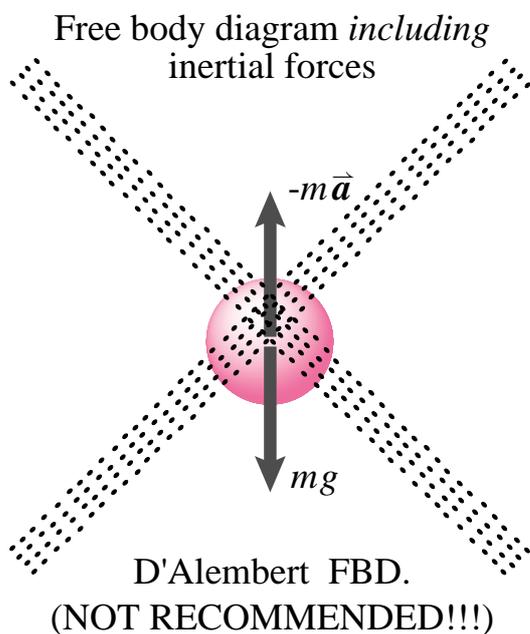
This box does not include any information needed for this course.

Many people believe that D'Alembert's approach to mechanics, an alternative to the momentum balance approach, should not be taught at this level. Students attempting to use D'Alembert methods make frequent mistakes. We do not advise the use of D'Alembert mechanics for first-time dynamics students.

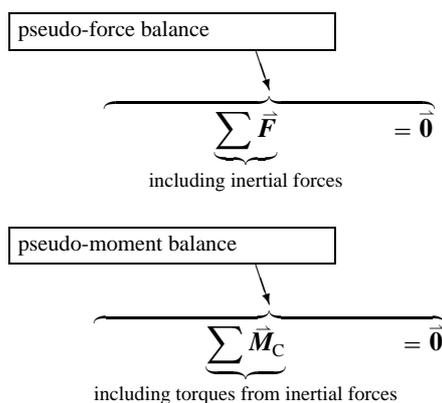
On the other hand, the D'Alembert approach has an intuitive appeal. Also, the D'Alembert equations are the first step in deriving the more advanced (*e.g.*, Lagrangian, Hamiltonian, 'method of virtual speed', and 'Kane') approaches to dynamics.

For completeness we briefly describe the approach.

First, label the free body diagram: '*free body diagram including inertial forces.*' Then, in addition to the applied forces draw pseudo-forces equal to $-m\vec{a}$ for every mass particle m . These pseudo-forces shown in the FBD of a falling ball using D'Alembert's approach to mechanics are sometimes called 'inertial' forces.



Instead of momentum balance equations you write 'pseudo-statics' equations of 'force' balance and 'moment' balance



These equations include the actual forces as well as the 'inertial' forces shown on the free body diagram.

By this means, the dynamics equations have been reduced to statics equations. Linear momentum balance is replaced by pseudo-statics force balance. Angular momentum balance is replaced by pseudo-statics moment balance.

The moving of the inertial terms from the right side of the equation to the left leads to both conceptual simplicity *and* puts the equations of dynamics in a form that is closer to most people's intuitions. The simplification is not so great as it may seem at first sight. Accelerations still need to be calculated and the sums involved in calculation of rate of change of linear and angular momentum still need to be calculated, only now they are sums of pseudo *inertial* forces.

Consider the example of sitting in a car as the car rounds a corner to the left. In the momentum balance approach, we write

$$\vec{F} = \underbrace{m\vec{a}}_{\vec{L}}$$

and say the force from the car on you to the left is equal to the rate of change of your linear momentum as you accelerate to the left. In the D'Alembert approach, we write

$$\vec{F} - \underbrace{m\vec{a}}_{\text{inertia force}} = \vec{0}$$

and think the inertia force to the right is balanced by the interaction force of the car on your body to the left.

It is a puzzle of human consciousness why such a trivial algebraic manipulation, namely,

$$\vec{F} = m\vec{a} \Rightarrow \vec{F} - m\vec{a} = \vec{0}$$

should lead to such a great conceptual confusion. But, it is an empirical fact that most of us are susceptible to this confusion.

That is, if you follow your likely first intuition and think of $m\vec{a}$ as a force you will probably join the ranks of many other talented students and make many sign errors.

Every teacher of mechanics has encountered the confusion in their students about whether $-m\vec{a}$ is or is not a force (and most likely in themselves as well.) To avoid such confusion, many teachers or texts take a firm stand and say

- ' $m\vec{a}$ is not a force!'; but, as if believing in a different god, others will say with equal conviction
- ' $-m\vec{a}$ is a force!'

In this book, we take the former approach. We take the equation

$$\vec{F} = m\vec{a}$$

to mean:

forces from interactions = $m \cdot$ (acceleration of mass).

If you insist on working with the D'Alembert approach instead, you must do so confidently and clearly. To repeat,

- instead of labeling your free body diagram '*FBD*', label it '*FBD including inertial forces*',
- instead of using '*Linear Momentum Balance*', use '*Pseudo-Force Balance*', and
- instead of using '*Angular Momentum Balance*' use '*Pseudo-Moment Balance*'

We do *not* recommend D'Alembert mechanics to beginners, but if you insist, good luck to you and don't blame us for your (almost inevitable) sign errors!