**Q: Derive a formula for the output signal *y* as a function of *t*.**

From the differential equation, I can say

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

If I let u(t) = $e^{\int 2dt} = e^{2t}$, and multiply both sides by u(t), I get

$$e^{2t}\frac{dy(t)}{dt} + 2e^{2t}y(t) = e^{2t}x(t)$$

Now, because $2e^{2t} = \frac{d}{dt}e^{2t}$ I get

$$e^{2t}\frac{dy(t)}{dt} + \frac{d}{dt}e^{2t}y(t) = e^{2t}x(t)$$

Applying the reverse product rule, I get

$$\frac{d}{dt}\left(e^{2t}y(t)\right)dt = e^{2t}x(t)$$

Now, integrating both sides

$$\int \frac{d}{dt}\left(e^{2t}y(t)\right)dt = \int e^{2t}x(t)dt \rightarrow e^{2t}y(t) = \int e^{2t}x(t)dt + c$$

Now, dividing both sides by u(t) gets me

$$y(t) = e^{-2t}\left(\int e^{2t}x(t)dt + c\right)$$

Which is the required solution.