

A study upon Eris

I. Describing and characterizing the orbit of Eris around the Sun

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Delivered March, 2013

ABSTRACT

Context. Describing and characterizing the orbit of Eris around the Sun.

Aims. Visualizing the orbit of Eris and characterizing it's physical quantities for each point of the trajectory.

Methods. To describe the motion of Eris around the Sun it was adopted a numerical approach. It was used the equation of the ellipse, the rotation matrices, the definitions of eccentric, true and mean anomalies and *Vis-viva equation*. In order to recover the characteristics of the elliptical orbit, using the data of velocity and position of one arbitrary point of the trajectory, it was used the specific orbital energy and specific angular momentum.

Results. The obtained results are accurate and consistent to the results obtained by NASA.

Key words. Keplerian elliptical orbit – dwarf planet Eris

1. Introduction

In the night of October 21, 2003, Eris was for the first time photographed. It was later discovered, on January 5 of 2005 in Palomar Observatory by the team lead by Mike Brown. By what is known, Eris is the most massive dwarf planet in the Solar System, and the ninth most massive body to orbit the Sun - it is estimated to have 1.01 the diameter^[1], and 1.27 the mass^[2] of Pluto. It is a trans-neptunian object, orbiting the Sun beyond the *Kuiper belt*, and a member of a high-eccentricity population known as the scattered disc. It has one known satellite named Dysnomia. Under the *International Astronomical Union* (IAU) definition approved on August 24, 2006, Eris is a "dwarf planet", along with objects such as Pluto, Ceres, Haumea and Makemake.

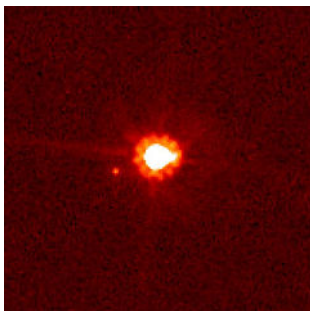


Fig. 1. Eris - center - and Dysnomia - left. Photography taken by *Hubble Space Telescope* in August 30, 2005.^[3]

It's aphelion – it's maximum distance from the Sun – it is estimated to be about 97.7 au. With the exception of some comets, Eris and Dysnomia are currently the most distant known natural objects in the Solar System.

This paper's objective is to describe and characterize the orbit of Eris around the Sun.

2. Adopted Methods and Procedures

The physical parameters necessary to characterize the movement of Eris around the Sun where collected from NASA's database and are presented in Table 1.

Table 1. Orbital Elements at 2013-April-18. TDB Reference: JPL 41 (heliocentric ecliptic J2000).^[4]

Element	Value	Uncertainty [1σ]	Units
e	0.4370836485780608	3.7521×10^{-5}	–
a	67.95782997290848	0.0032706	au
perihelion	38.25457369890214	0.0043578	au
aphelion	97.66108624691483	0.0047002	au
i	43.88534914515534	0.0004675	deg
Ω	36.03089590229352	0.00025694	deg
ω	150.8002699050311	0.0056012	deg
period	560.23	0.04044	yr
mass ^[5]	8.35×10^{-9}	1.0×10^{-10}	M_{\odot}

2.1. Describing the Orbit of Eris

– Drawing the orbit

The orbit of Eris around the Sun is described by an ellipse with the characteristics presented in Table 1. The equation of the ellipse is given by:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1, \quad (1)$$

$$b = a \sqrt{1 - e^2}, \quad (2)$$

where x and y are the cartesian coordinates, a the semi-major axis, b the semi-minor axis and e the eccentricity.

One way to describe the x and y coordinates along the orbit, is to parameterize by angle:

$$\begin{aligned} x_i &= a \cos(\theta_i), \\ y_i &= b \sin(\theta_i), \end{aligned} \quad (3)$$

where $-\pi \leq \theta \leq \pi$.

As a first step in the description of the orbit of Eris, the adopted method consists in a numerical approximation, by solving equation 3 for a discretized interval of values for θ . As in all numerical approach, the smaller the step of the discretization, the higher the precision of the calculus.

With the data presented in Table 1, by solving equation 3, it is possible to describe the orbit of Eris in two dimensions. In order to simulate the true orbit of Eris around the Sun it is necessary to rotate the obtained ellipse in x and z . The rotations will dispose the orbit in the right position according to the ecliptic.

The rotation matrices are given by:

$$R_z(\omega) = \begin{bmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (4)$$

$$R_x(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & \sin(i) \\ 0 & -\sin(i) & \cos(i) \end{bmatrix}, \quad (5)$$

$$R_z'(\Omega) = \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (6)$$

where ω is the argument of perihelion, i the inclination and Ω the longitude of the ascending node.

By applying the rotation matrices, it is obtained the new position of the orbit of Eris, according to the ecliptic.

Since the Sun is in one of the two focus of the ellipse, it is necessary to transpose the center of the coordinate system to the Sun's position. In the 2D projection of the orbit, the coordinates of the Sun are:

$$\begin{aligned} x_{sun} &= a - \text{perihelion}, \\ y_{sun} &= 0, \\ z_{sun} &= 0. \end{aligned}$$

By applying the same transformations – rotations – to this specific point, it is possible to calculate the new position of the Sun. In order to obtain the 3D orbit of Eris, with the Sun in it's

right position, being the center of the coordinate system, it is necessary to add the obtained values for the coordinates of the Sun to the previously computed coordinates that describe the rotated ellipse:

$$\begin{aligned} x'_{eris,i} &= x_{eris,i} + x'_{sun}, \\ y'_{eris,i} &= y_{eris,i} + y'_{sun}, \\ z'_{eris,i} &= z_{eris,i} + z'_{sun}. \end{aligned}$$

– *Distance to the Sun along the orbit*

To calculate the distance between the Sun and Eris, r , along the orbit:

$$r_i = \sqrt{x_i^2 + y_i^2 + z_i^2}. \quad (7)$$

The values of r_i for the orbits of Eris in 2D and 3D – both with the center in the focus – must be equal for the projection to be correct, since the module of the position coordinates are conserved in the process of rotation.

– *Velocity*

The *Vis-viva equation*, also referred as *orbital energy conservation equation*, arises as a consequence of the conservation of energy, which requires the sum of the kinetic and potential energy of the body along the orbit to be constant. This equation relates the velocity of the body along the orbit with it's distance to the center of mass, and is given by:

$$v_i^2 = \mu \left(\frac{2}{r_i} - \frac{1}{a} \right), \quad (8)$$

$$\mu = GM, \quad (9)$$

where v_i is the module of the velocity along the orbit, μ the *standard gravitational parameter*, G the *gravitational constant* and M the total mass of the physical system.

It is also possible to compute the velocity components by verifying that the derivative of the ellipse – dy/dx – is the value of the slope of the tangent line – velocity vector – in each point. Having the slope and the module of the velocity, it is possible to characterize the velocity in it's x and y components.

The derivative of the ellipse along the curve is given by:

$$y'_i = \left| -\frac{x_i b^2}{y_i a^2} \right|. \quad (10)$$

The angle between the velocity vector and the coordinate axes can be calculated by:

$$\alpha_i = \arctan(y'_i). \quad (11)$$

The velocity components are calculated by:

$$\begin{aligned} v_{x,i} &= c_1 v_i \cos(\alpha_i), \\ v_{y,i} &= c_2 v_i \sin(\alpha_i), \end{aligned} \quad (12)$$

where $v_{x,i}$ and $v_{y,i}$ are the velocity x and y components along the orbit, respectively. The constants c_1 and c_2 can be 1 or -1, as it is shown in Figure 1.

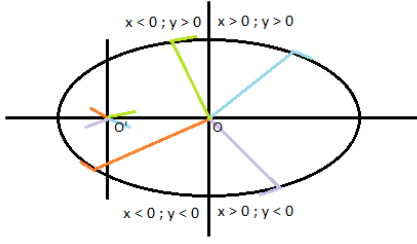


Fig. 2. Schematic representation of the distance between the center of the ellipse and the orbiting object, and correspondent velocity along the orbit. By analysing the figure, it is verified that when the object is moving inside one of the 4 quadrants, the projection of the vector in the focus will correspond to the next quadrant. This way, when the components of the position (A) belong to $x < 0 ; y > 0$ the velocity components (B) behave in a way that $x' > 0 ; y' < 0$ (where x', y' refers to the coordinate system S' , with center in O'). So, for A: $x < 0 ; y > 0$, $c_1 = 1$ $c_2 = 1$. For A: $x > 0 ; y < 0$, $c_1 = 1 ; c_2 = -1$. For A: $x > 0 ; y < 0$, $c_1 = -1 ; c_2 = -1$. For A: $x < 0 ; y < 0$, $c_1 = -1 ; c_2 = 1$.

– Eccentric Anomaly

The *eccentric anomaly*, E , is an angular parameter that defines the position of a body that is moving along an elliptic *Keplerian* orbit. It is the angle between the center of the ellipse and the projection in y of the position of the orbiting body in an circle with radius equal to the semi-major axis of the ellipse centered in the center of the ellipse, as it is shown in Figure 3.

The eccentric anomaly is given by:

$$E_i = \arccos\left(\frac{1 - r_i/a}{e}\right). \quad (13)$$

– True Anomaly

As the eccentric anomaly, the *true anomaly*, v , is another angular parameter that defines the position of a body moving along a *Keplerian* orbit. Is defined as the angle between the direction of the perihelion and the position of the object, and is given by:

$$v_i = 2 \arctan\left(\sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E_i}{2}\right)\right). \quad (14)$$

– Mean Anomaly

The time the body takes to move along the orbit, since the perihelion passage, can be estimated by an parameter known as the *mean anomaly*, M . This parameter relates position and time of a body that is moving along an elliptic *Keplerian* orbit. It arises as a consequence of the *Kepler's 2nd law* which stands that equal areas are swept in equal intervals of time by a line joining the focus and the orbiting body. This quantity can be expressed as:

$$M_i = E_i - e \sin(E_i), \quad (15)$$

or

$$M_i = n t_i = \sqrt{\frac{\mu}{a^3}} t_i, \quad (16)$$

where n is known as the *mean motion*.

With these two relations it is possible to describe the time along the orbit, t_i , which is the time since the body leaves the perihelion.

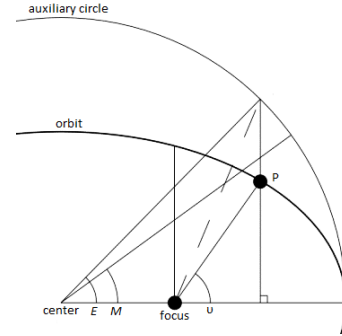


Fig. 3. Schematic representation of the eccentric, mean and true anomalies. [6].

2.2. Characterizing the orbit from velocity and position of an arbitrary point

With the velocity and position components of a single point of the trajectory, it is possible to characterize the ellipse that defines the orbit, which means, to recover the values of eccentricity, semi-major axis and semi-minor axis.

– Semi-major axis

This quantity can be obtained solving (8) in order to the semi-major axis, a . Solving for all the points of the trajectory, the values a_i are obtained.

– Eccentricity

One possible way to obtain the value of eccentricity using the values of velocity and position for one point of the trajectory, is to calculate the *specific orbital energy* and the *specific angular momentum*. These are conservative quantities, so they must remain the same in every point, for the calculations to be correct.

The specific orbital energy, ϵ , is the sum of the potential and kinetic energies of the system, divided by the total mass. It can be written as:

$$\epsilon_i = \frac{v_i^2}{2} - \frac{\mu}{r_i}. \quad (17)$$

As for the specific angular momentum, h , is the vector product of position and velocity:

$$h_i = r \times v = r_{x,i}v_{y,i} - r_{y,i}v_{x,i}. \quad (18)$$

The eccentricity can then be calculated through:

$$e_i = \sqrt{1 + \frac{2\epsilon_i h_i^2}{\mu^2}} \quad (19)$$

– Semi-minor Axis

Since the semi-minor axis, b , is related with the semi-major axis and eccentricity, its value can be obtained solving (2). By solving for all the points of the trajectory, the values b_i are obtained.

To verify if the previous calculations are correct, the obtained values for semi-major axis, eccentricity and semi-minor axis must be equal for every point of the trajectory and must correspond to the values acquired by *NASA*, presented in Table 1.

3. Results

– Orbit of Eris

The next Figures will present the results obtained for the orbit of Eris in two and in three dimensions.

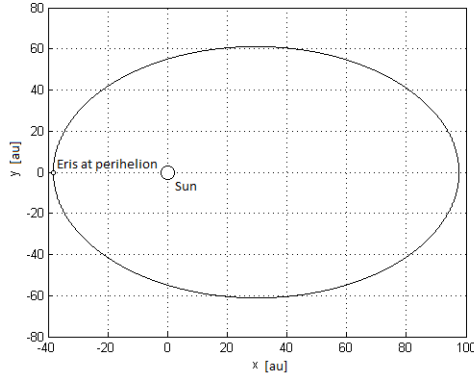


Fig. 4. Representation of the orbit of Eris around the Sun in two dimensions.

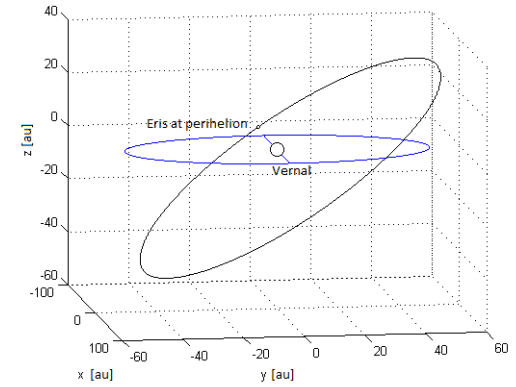
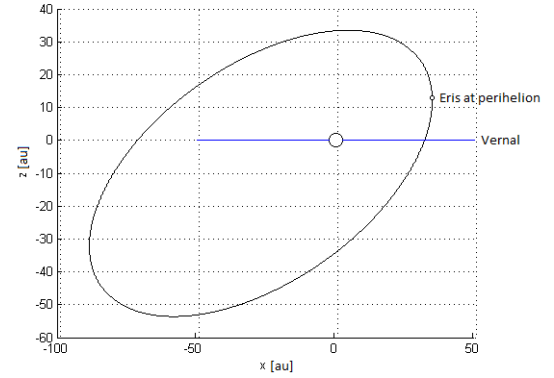
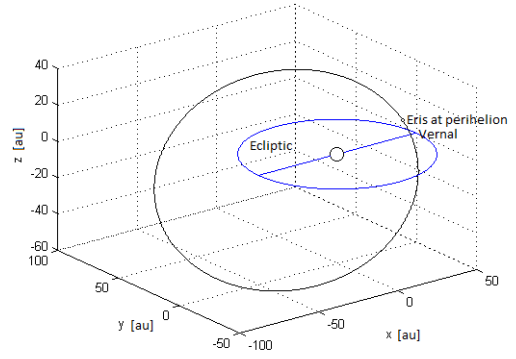


Fig. 5. Representation of the orbit of Eris around the Sun in three dimensions, through three different perspectives.

It was confirmed that in every point of the trajectory, the value of r_i for both projections is the same.

With the values presented in Table 1, following the procedures explained above, the values of t_i , v_i , $r_{x,i}$, $r_{y,i}$, $r_{z,i}$, v_i and r_i were calculated, with a numerical step of $\delta\theta = 1.309 \times 10^{-4}$. Table 2 presents these physical quantities, showing the results every $i = 1 + n * 800$; $0 \leq n \leq 60$, starting at the perihelion.

As for Table 3, it presents the values for semi-major axis, eccentricity and semi-minor axis computed with the components of position and velocity, for the same points of the trajectory as Table 2.

Table 2. Results obtained for the orbit of Eris around the Sun, for several values of θ .The values were computed for a numerical step $\delta\theta = 1.309 \times 10^{-4}$, showing the results every $i = 1 + n * 800$; $0 \leq n \leq 60$.

<i>Absolute Time</i> [yr]	<i>True Anomaly</i> [π rad]	r_x [au]	r_y [au]	r_z [au]	<i>Velocity</i> [au/s]	<i>Distance to the Sun</i> [au]
7.48 $\times 10^{-7}$	2.38 $\times 10^{-8}$	34.91723	-8.76507	12.93733	3.85 $\times 10^{-8}$	38.25459
5.26363	0.16709	34.73366	-13.76399	8.94515	3.84 $\times 10^{-8}$	38.41732
10.57188	0.33277	33.87247	-18.53754	4.74490	3.81 $\times 10^{-8}$	38.90371
15.96893	0.49573	32.34309	-23.03341	0.38260	3.75 $\times 10^{-8}$	39.70843
21.49792	0.65481	30.16228	-27.20234	-4.09396	3.68 $\times 10^{-8}$	40.82268
27.20059	0.80905	27.35394	-30.99866	-8.63573	3.59 $\times 10^{-8}$	42.23423
33.11677	0.95772	23.94883	-34.38077	-13.19293	3.49 $\times 10^{-8}$	43.92764
39.28395	1.10035	19.98427	-37.31161	-17.71566	3.38 $\times 10^{-8}$	45.88433
45.73684	1.23667	15.50369	-39.75907	-22.15434	3.26 $\times 10^{-8}$	48.08287
52.50707	1.36660	10.55619	-41.69633	-26.46034	3.14 $\times 10^{-8}$	50.49918
59.62275	1.49022	5.19597	-43.10218	-30.58650	3.01 $\times 10^{-8}$	53.10678
67.10823	1.60774	-0.51824	-43.96120	-34.48759	2.89 $\times 10^{-8}$	55.87709
74.98381	1.71944	-6.52382	-44.26399	-38.12087	2.76 $\times 10^{-8}$	58.77977
83.26549	1.82565	-12.75498	-44.00722	-41.44655	2.64 $\times 10^{-8}$	61.78301
91.96484	1.92677	-19.14345	-43.19371	-44.42816	2.53 $\times 10^{-8}$	64.85391
101.08890	2.02318	-25.61922	-41.83237	-47.03306	2.41 $\times 10^{-8}$	67.95882
110.63990	2.11528	-32.11135	-39.93812	-49.23270	2.30 $\times 10^{-8}$	71.06371
120.61560	2.20345	-38.54870	-37.53171	-51.00297	2.20 $\times 10^{-8}$	74.13457
131.00890	2.28808	-44.86074	-34.63952	-52.32449	2.11 $\times 10^{-8}$	77.13776
141.80840	2.36953	-50.97832	-31.29322	-53.18276	2.02 $\times 10^{-8}$	80.04037
152.99790	2.44814	-56.83440	-27.52948	-53.56840	1.93 $\times 10^{-8}$	82.81059
164.55720	2.52423	-62.36482	-23.38955	-53.47716	1.85 $\times 10^{-8}$	85.41808
176.46200	2.59811	-67.50898	-18.91877	-52.91006	1.78 $\times 10^{-8}$	87.83426
188.68400	2.67007	-72.21053	-14.16614	-51.87330	1.72 $\times 10^{-8}$	90.03266
201.19180	2.74039	-76.41795	-9.18373	-50.37825	1.67 $\times 10^{-8}$	91.98919
213.95060	2.80932	-80.08513	-4.02613	-48.44129	1.62 $\times 10^{-8}$	93.68242
226.92280	2.87712	-83.17191	1.25016	-46.08363	1.58 $\times 10^{-8}$	95.09380
240.06860	2.94402	-85.64446	6.58731	-43.33111	1.55 $\times 10^{-8}$	96.20786
253.34650	3.01026	-87.47568	11.92686	-40.21390	1.53 $\times 10^{-8}$	97.01238
266.71300	3.07606	-88.64552	17.21030	-36.76613	1.51 $\times 10^{-8}$	97.49857
280.10740	3.14155	-89.14116	22.37974	-33.02560	1.51 $\times 10^{-8}$	97.66109
293.51850	3.07597	-88.95716	27.37854	-29.03328	1.51 $\times 10^{-8}$	97.49817
306.88500	3.01018	-88.09555	32.15193	-24.83291	1.53 $\times 10^{-8}$	97.01158
320.16270	2.94394	-86.56576	36.64761	-20.47052	1.55 $\times 10^{-8}$	96.20665
333.30840	2.87703	-84.38455	40.81632	-15.99391	1.58 $\times 10^{-8}$	95.09222
346.28030	2.80924	-81.57583	44.61240	-11.45212	1.62 $\times 10^{-8}$	93.68048
359.03880	2.74030	-78.17036	47.99423	-6.89492	1.67 $\times 10^{-8}$	91.98691
371.54620	2.66998	-74.20546	50.92478	-2.37223	1.72 $\times 10^{-8}$	90.03006
383.76790	2.59802	-69.72458	53.37193	2.06638	1.78 $\times 10^{-8}$	87.83137
395.67230	2.52413	-64.77680	55.30887	6.37229	1.85 $\times 10^{-8}$	85.41493
407.23110	2.44804	-59.41634	56.71438	10.49832	1.93 $\times 10^{-8}$	82.80722
418.42020	2.36943	-53.70193	57.57305	14.39925	2.02 $\times 10^{-8}$	80.03682
429.21910	2.28798	-47.69618	57.87549	18.03236	2.11 $\times 10^{-8}$	77.13406
439.61200	2.20334	-41.46490	57.61837	21.35783	2.20 $\times 10^{-8}$	74.13077
449.58710	2.11516	-35.07636	56.80451	24.33922	2.30 $\times 10^{-8}$	71.05984
459.13760	2.02306	-28.60055	55.44284	26.94387	2.41 $\times 10^{-8}$	67.95493
468.26110	1.92665	-22.10844	53.54826	29.14325	2.53 $\times 10^{-8}$	64.85004
476.95990	1.82552	-15.67114	51.14154	30.91325	2.64 $\times 10^{-8}$	61.77921
485.24110	1.71930	-9.35920	48.24905	32.23447	2.76 $\times 10^{-8}$	58.77608
493.11620	1.60760	-3.24177	44.90248	33.09246	2.89 $\times 10^{-8}$	55.87354
500.60120	1.49007	2.61413	41.13850	33.47779	3.01 $\times 10^{-8}$	53.10341
507.71640	1.36644	8.14432	36.99834	33.38626	3.14 $\times 10^{-8}$	50.49604
514.48620	1.23650	13.28823	32.52737	32.81886	3.26 $\times 10^{-8}$	48.07998
520.93870	1.10018	17.98948	27.77458	31.78182	3.38 $\times 10^{-8}$	45.88173
527.10560	0.95754	22.19658	22.79204	30.28648	3.49 $\times 10^{-8}$	43.92535
533.02150	0.80886	25.86341	17.63435	28.34925	3.59 $\times 10^{-8}$	42.23229
538.72390	0.65461	28.94982	12.35801	25.99134	3.68 $\times 10^{-8}$	40.82110
544.25270	0.49553	31.42197	7.02083	23.23858	3.75 $\times 10^{-8}$	39.70723
549.64960	0.33256	33.25279	1.68130	20.12115	3.81 $\times 10^{-8}$	38.90290
554.95780	0.16688	34.42221	-3.60208	16.67319	3.84 $\times 10^{-8}$	38.41691
560.21480	2.38 $\times 10^{-8}$	34.91723	-8.76507	12.93733	3.85 $\times 10^{-8}$	38.25459

By examine Table 3, it can be concluded that the ellipse parameters obtained are equal to NASA's values, with a maximum error of the order of 10^{-15} , which means that the adopted procedures provide very accurate results.

4. Discussion and Details

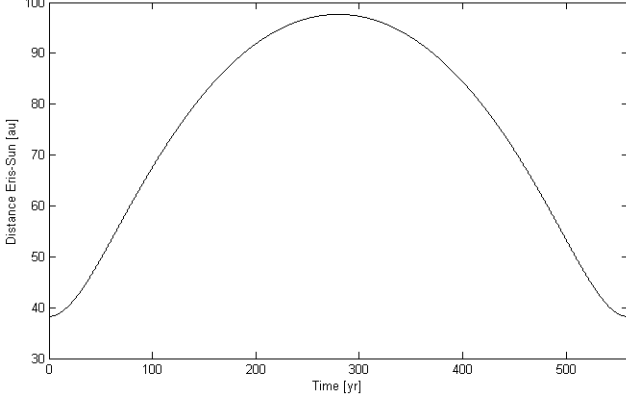


Fig. 6. Distance of Eris relatively to the Sun trough time.

In Figure 6 it can be visualized the distance between Eris and the Sun trough time. The maximum value – aphelion distance – is reached when the time is half of the period.

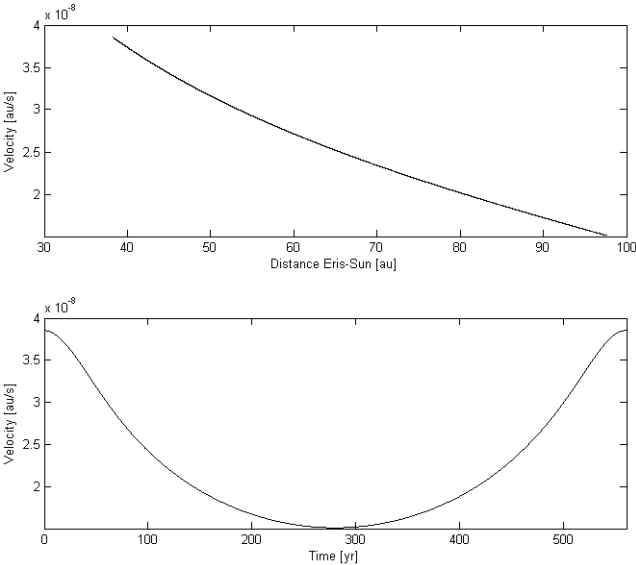


Fig. 7. Velocity of Eris relatively to the distance Eris-Sun (above) and Velocity of Eris trough time (below).

By analysing Figure 7 (above) it can be verified that Eris reaches it's maximum velocity when it's distance to the Sun it's minimal. This result was expected since when the distance between both objects is minimal, the attraction force – which is negative – is maximum, so the orbiting body must increase it's kinetic energy – velocity – in order to conserve the total energy of the system.

Figure 7 (below) shows that the velocity reaches its maximum in the beginning and in the end of the calculation – when it's closer to the Sun – and reaches it's minimum at half the total time of the calculation which corresponds to the aphelion

– point when it's farther to the Sun.

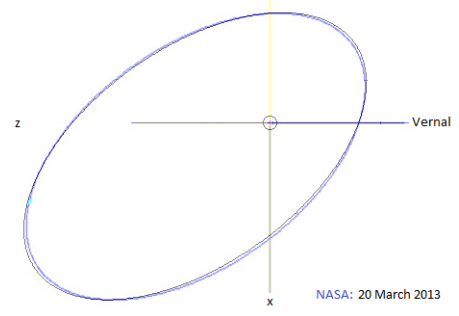


Fig. 8. Comparing the results obtained for the orbit of Eris – black line – with the orbit of Eris from NASA – blue line – for March, 20 of 2013, day where Earth is in it's vernal point.

In an attempt of comparing the obtained rotated orbit with the results of NASA, it was used a rudimentary way to overlap both ellipses. It can be seen small deviations, but probably they are present due to the flaws of the rustic procedure adopted to overlap the two ellipses. Analysing Figure 8 it is possible to conclude that the rotations applied are correct.

– If Eris initially had a circular orbit around the Sun, with $r = \text{perihelion}$, how much would be the difference in linear velocity needed for Eris to have this elliptical orbit?

To answer this question it was chosen an *Newtonian* approach.

"To every action there is always an equal and opposite reaction: or the forces of two bodies on each other are always equal and are directed in opposite directions."

$$F_{i,j} = -F_{j,i}$$

When one object experience a stable orbit around another, it means that they are in equilibrium of force – the object that orbits around the other has a centripetal acceleration that sustains the circular orbit. This way, it is possible to derive the velocity of the first object from the system of equations for the force.

Since:

$$\bar{m}_j \bar{a}_j = G m_i m_j \frac{\bar{r}_i - \bar{r}_j}{|\bar{r}_i - \bar{r}_j|^3}, \quad (20)$$

$$\bar{a}_c = \frac{\bar{v}^2}{\bar{r}_i - \bar{r}_j}, \quad (21)$$

The module of the velocity of the object that experiences the orbit is:

$$v_j = \sqrt{\frac{\mu}{r}}, \quad (22)$$

where r is the distance between the two bodies.

By solving (22) with $r = \text{perihelion}$, it is computed the velocity of Eris assuming a circular orbit – $v_c = 3.2151 \times 10^{-8}$.

Using (8) with $r_i = \text{perihelion}$, it is computed the velocity of Eris in the perihelion – $v_p = 3.8542 \times 10^{-8}$ – assuming it's elliptical orbit. By subtracting the two values, the obtained result was of $v = 6.391145 \times 10^{-9}$ [au/s].

In order to confirm this result, it was used the *N-body Simulator* developed in previous works.

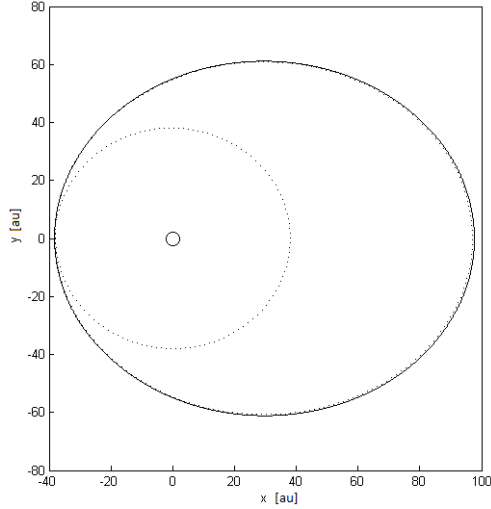


Fig. 9. Orbit of Eris obtained with *N-body simulator* – dashed line – and with the procedures explained above – solid line. The circular dashed line was obtained with the *N-body simulator* using a initial velocity equal to v_c and the elliptical dashed line using a initial velocity of v_p .

Figure 9 presents the results of the *N-body simulator* – dashed line – and the results obtained with the procedures explained above – solid line. By giving the simulator the value for initial velocity v_c it simulates a circular orbit around the Sun. Giving the value of v_p as initial velocity, the result is very close from the orbit of Eris computed previously, which permits to conclude that the *Newtonian* approach used to calculate the difference of velocities was successful. The small deviations of the elliptical dashed orbit and elliptical solid orbit can be explained by the numerical error committed in the simulation, or, due to the fact that large objects like Jupiter and Saturn – that have enough mass to interact smoothly with Eris – are not considered in the simulation.

5. Conclusions

By analysing the results it can be concluded that the adopted procedures provide accurate results, since they allow to determine correctly the parameters of the orbit of Eris.

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