

# Nyquist Stability

## Aim:

Apply Nyquist stability criterion (NSC) to design PID controller for airflow heating process

## The Equipment:

Process Trainer 37-100



# Objectives (1)

**Task 1:** Identify the open-loop system transfer function

**Task 2:** Find model-based tuned PI controller gains for the process, using

a) Ziegler-Nichols tuning formulae

b) Cohen and Coon tuning formulae

**Task 3:** Find model-based tuned PI controller gains for the process, using Nyquist gain margin criterion in Matlab

# Objectives (2)

**Task 4:** Investigate the effect of changing the gain margin on the closed loop step response in Matlab

**Task 5:** Implement the three model-tuned PI controllers from Tasks 2 and 3 on the Process Trainer. Then carry out manual fine-tuning of the controller from Task 2.

# The Process

Air is circulated along a polypropylene tube by an axial fan.

A heating element controlled by a thyristor circuit heats the airstream.

A thermistor detector senses the temperature - the detector's position along the tube determines the transportation lag in the process.

The volume of air flow is controlled by varying the speed of the fan via a potentiometer. A change in setting represents a supply side disturbance

# Process Trainer Control Computer

The process is controlled by a digital PID controller which is resident in the Process Trainer Control computer.

The Process Trainer Control computer consists of a white system unit and white operator interface (monitor and keyboard). On operator keyboard:

- Keys 'O' and 'C' are used to change to open-loop or closed-loop operation of the Process Trainer
- Key 'K' is used to change the PID controller gains
- Key 'P' is used to print the process time responses displayed on the monitor

# Preparation for Matlab Simulations

Simulations of the process control system will be carried out using the MATLAB package which is resident in the Process Trainer Simulation computer.

The simulation computer consists of a black system unit, monitor and keyboard. Prepare for the simulations to be conducted in Tasks 2, 3 and 4 by following the steps in slide 6.

# Prepare for Simulations

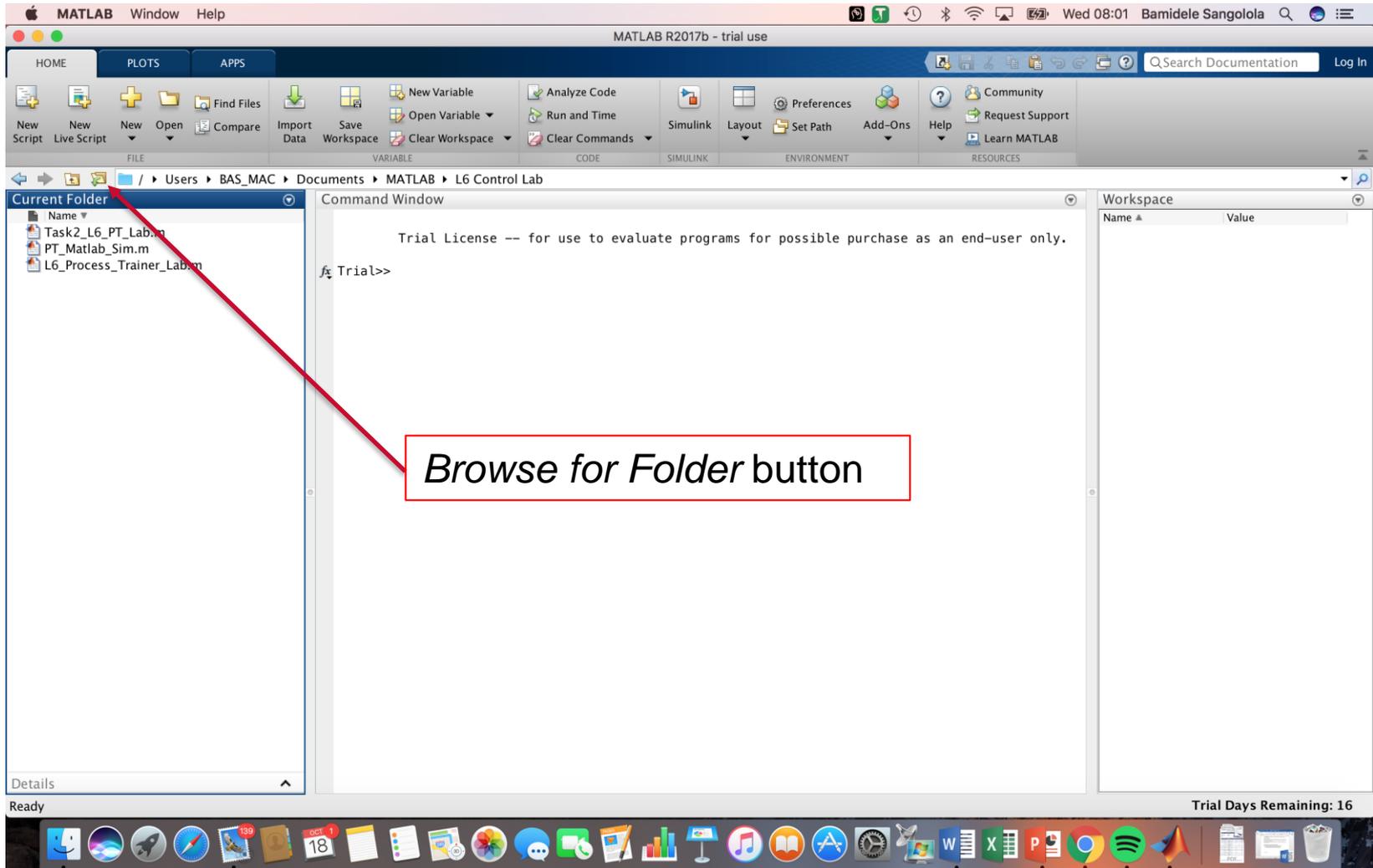
- Start MATLAB and use the *Browse for Folder* button (see the screen shot in the next slide) to change the Current Folder to the ***Documents/My Documents/MATLAB*** folder in ***Libraries***.
- Login to your Flight Systems or Control course on blackboard and navigate to **Learning Materials** section.

# Prepare for Simulations

- Open the folder named **Control Lab**. In the folder, an item named **Matlab files** has the following **four** attachments: *Task2\_L6\_PT\_Lab.m*, *Task3\_L6\_PT\_Lab.m*, *Task4\_L6\_PT\_Lab.m*, and *PT\_Matlab\_Sim.m*
- Right click on each of the four files in the folder, select the *Save Link As...* option, and save to the ***Documents/ My Documents/MATLAB*** folder in ***Libraries***.
- If the four files you copied can be seen in the MATLAB Current Folder, you have successfully completed the necessary preparation to use MATLAB for Tasks 2, 3, and 4.

# Prepare for Simulations

## Changing the Current Folder in Matlab



# Process Model

The process (i.e., open-loop system transfer function) can be represented by the following transfer function

$$G_p(s) = \frac{Ke^{-t_d s}}{1 + t_p s}$$

where

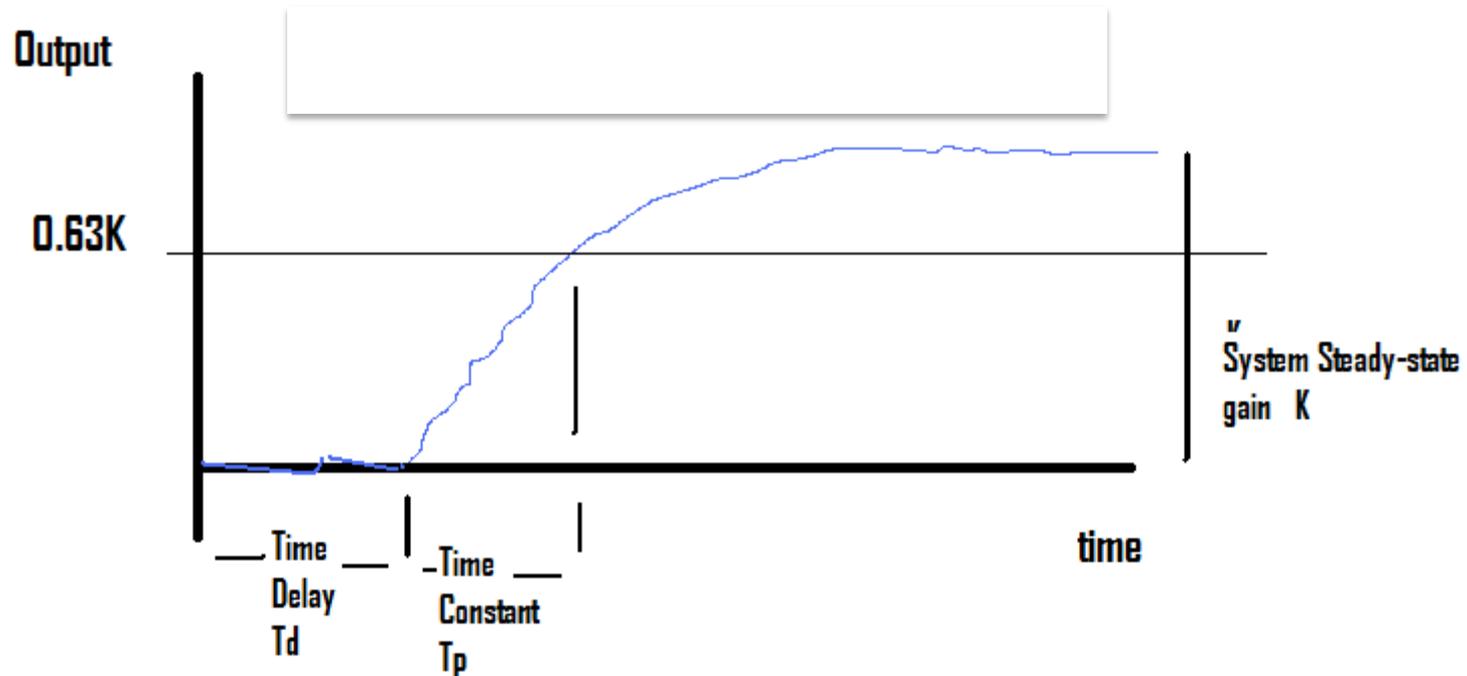
$K$  is steady state gain

$t_p$  is the system time constant

$t_d$  is the system time delay

# Process Reaction Curve

An open-loop step response of the system is shown below



# Closed Loop PI Control

For a Proportional + Integral (PI) control, the controller transfer function is

$$G_c(s) = \frac{u(s)}{e(s)} = K_p + \frac{K_i}{s} = K_p \left( 1 + \frac{1}{T_i s} \right)$$

where

$e(s)$  is the Laplace transformed error signal

$u(s)$  is the Laplace transformed controller output

$K_p$  is the proportional gain,

$K_i$  is the integral gain,

$T_i$  is the integral time.

# Nyquist Stability Criterion (1)

Statement of the criterion:

$$N = P - Z$$

where

- $P$  is the number of open-loop unstable poles
- $Z$  is the number of closed-loop unstable poles
- $N$  is the number of anticlockwise encirclements of the  $(-1,0)$  point by the open-loop Nyquist plot

For an open-loop stable system ( $P = 0$ ), the closed-loop system is stable ( $Z = 0$ ) if  $N = 0$ .

## Nyquist Stability Criterion (2)

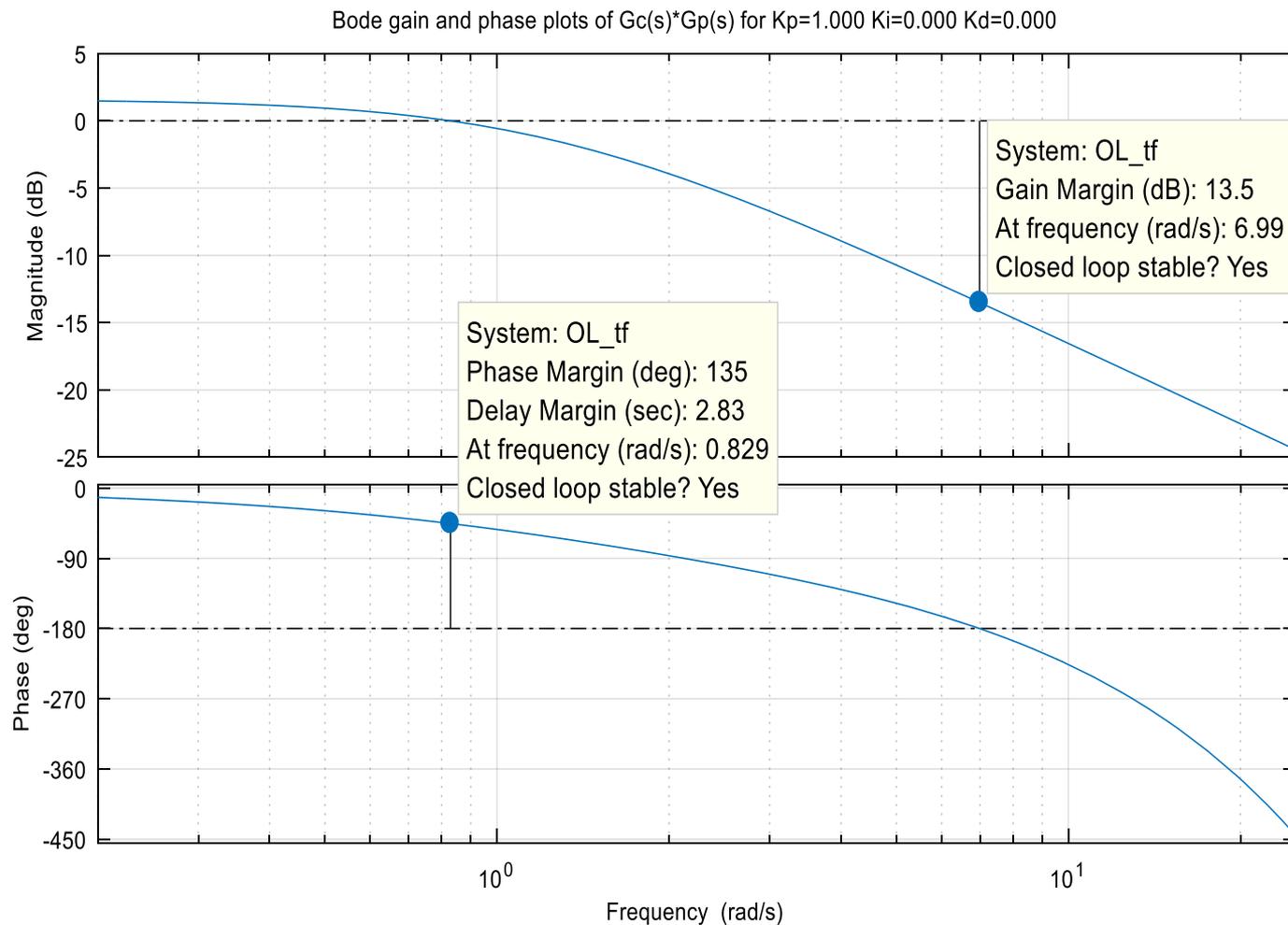
Applying the NSC to the Process Trainer :

$$P = 0$$

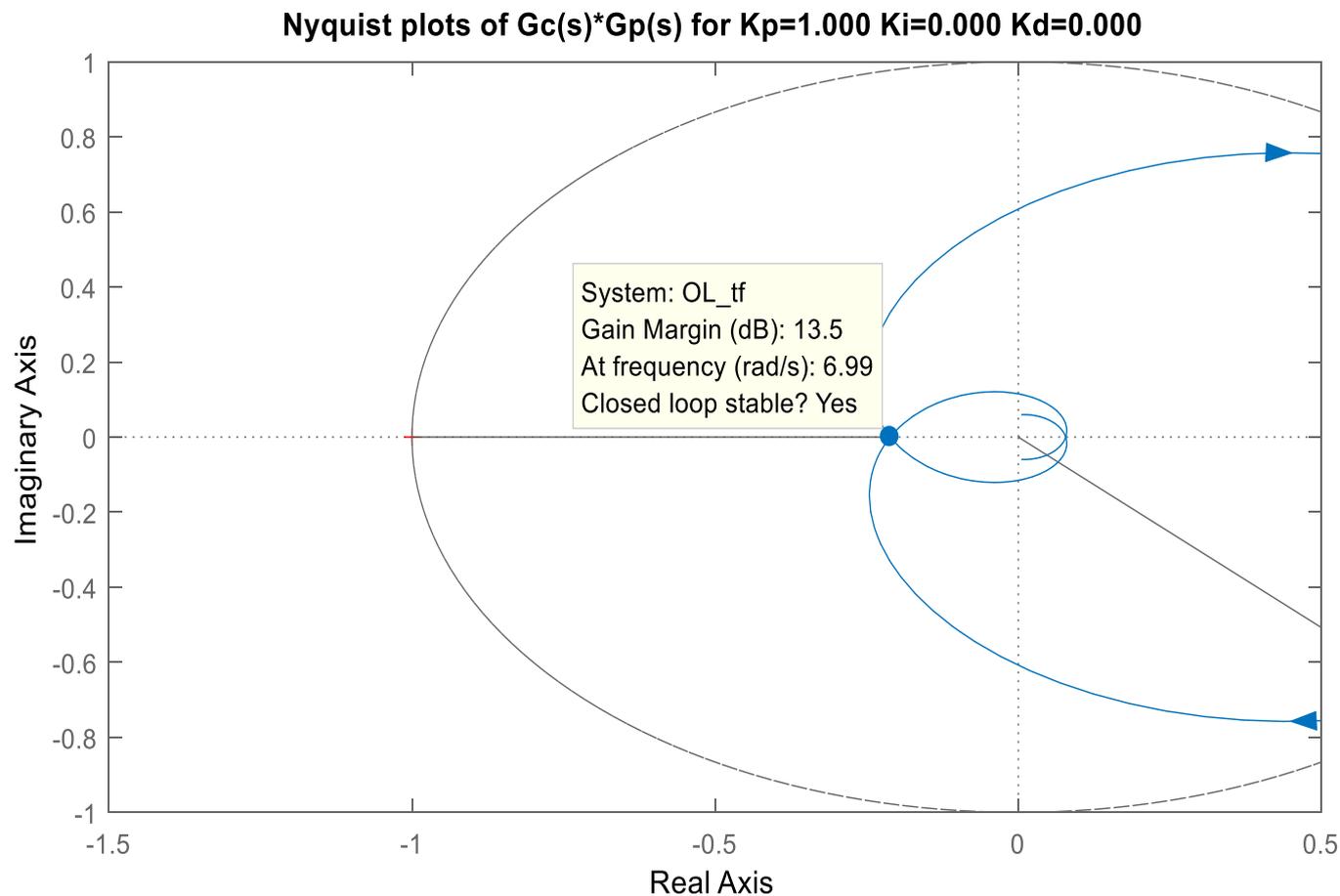
Therefore, for  $Z = 0$  we require that  $N = 0$ . This means that the OL Nyquist plot for the Process Trainer must not encircle the  $(-1,0)$  point.

The gain margin,  $GM$ , and phase margin,  $PM$ , are open-loop frequency response characteristics used to assess how close a closed-loop system is to instability.

# Bode Gain and Phase Margins

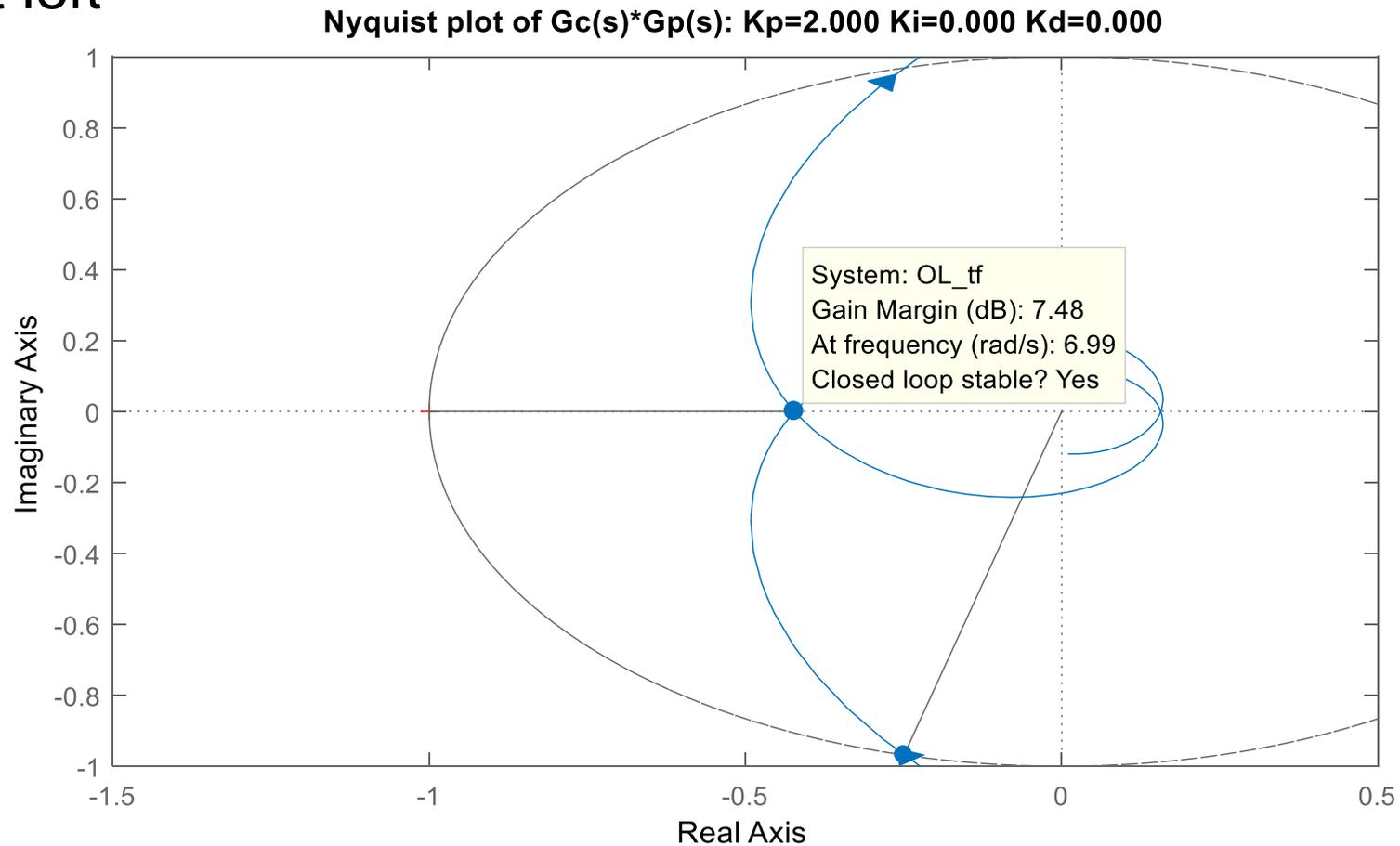


# Nyquist Gain and Phase Margins



# Nyquist Plot Gain Margin Adjustment (1)

Increase in  $K_p$  shifts the real axis intercept of the Nyquist plot left



# Nyquist Plot Gain Margin Adjustment (2)

If

$$K_{p,1} \rightarrow GM1$$

and

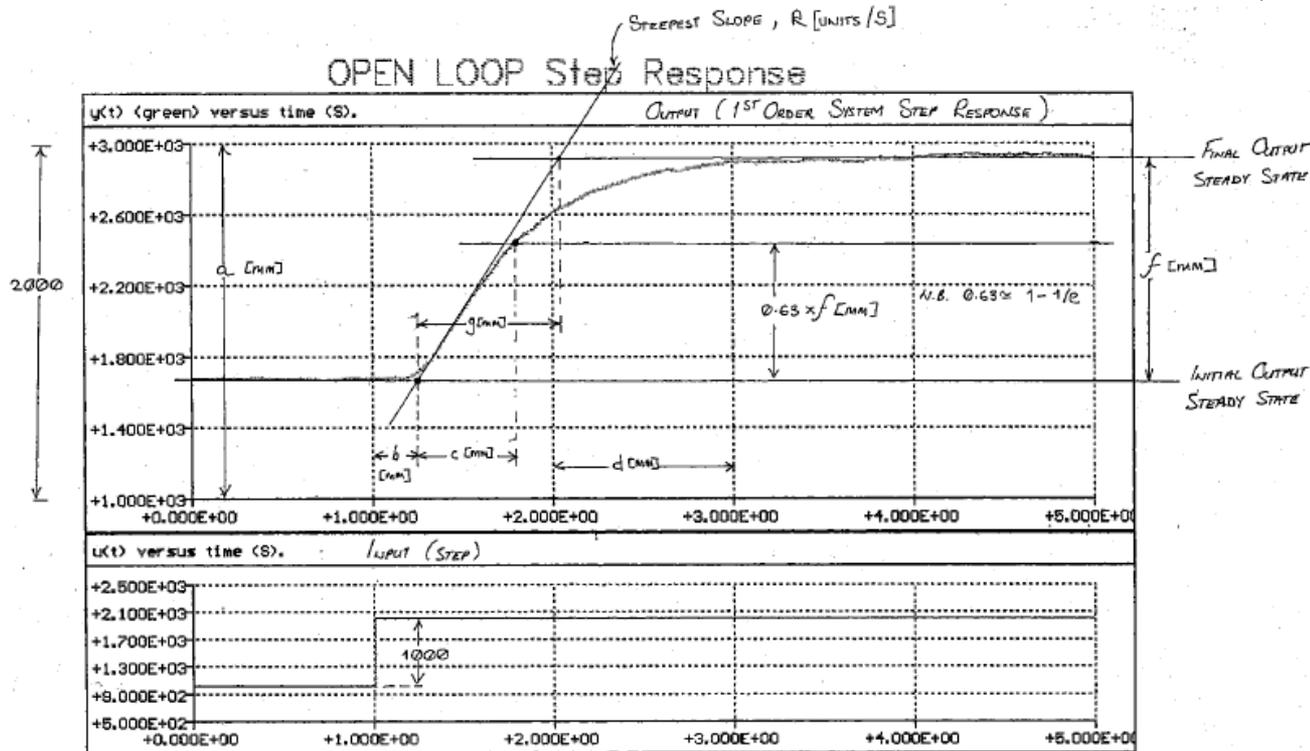
$$K_{p,2} \rightarrow GM2$$

then

$$K_{p,2} = K_{p,1} \times \frac{GM1}{GM2}$$

# Task1: Open-Loop System Identification

Obtain the open loop step response of the process. Use the measurements shown below to work out  $K$ ,  $T_p$  and  $T_d$



$$D.C. \text{ GAIN, } K = \frac{f \times 2000}{\frac{a}{1000}} = \frac{2f}{a} \quad \text{TIME CONSTANT, } T_p = \frac{c}{d} \text{ [S]} \quad \text{TIME DELAY, } T_d = \frac{b}{d} \text{ [S]}$$

$$R = \frac{f}{\frac{a}{g}} \times 2000 = \frac{1000 \times K \times d}{g} \text{ [UNITS/S]}$$

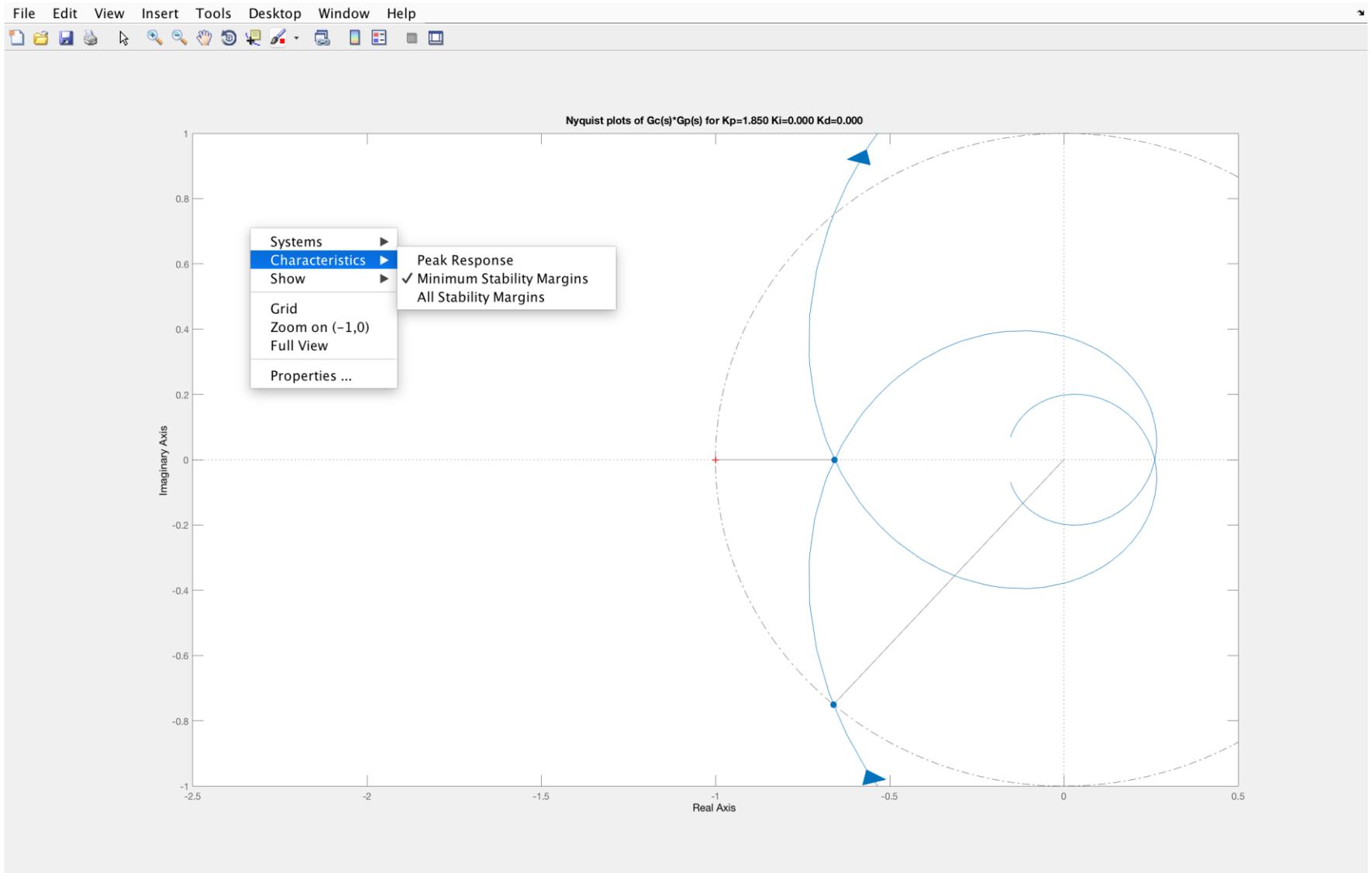
# Saving Results for Tasks 2 & 3 (1)

- The procedures for Tasks 2, 3 and 4 are described in slides 26-31. Each simulation run in MATLAB will produce a Nyquist plot, a Bode plot, and a step response plot
- On each Nyquist plot, right click inside the plot to annotate the plot with  $GM$  and  $PM$  values as illustrated in slide 22
- On each Bode plot, right click inside the plot to annotate the plot with  $GM$  and  $PM$  values as illustrated in slide 23

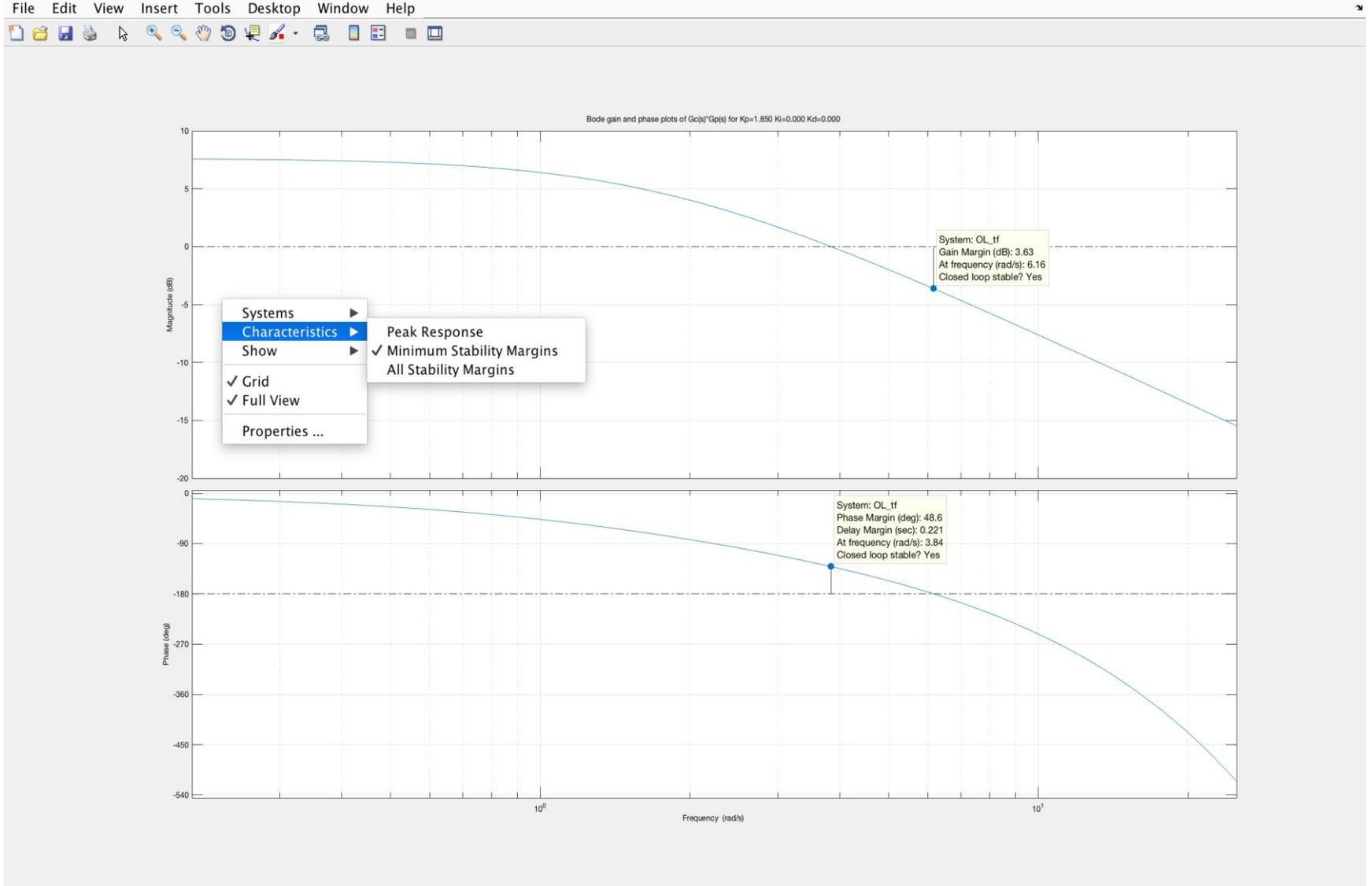
# Saving Results for Tasks 2 & 3 (2)

- On each step response plot, right click inside the plot to annotate the plot with  $M_p$ ,  $t_s$  and  $y_{ss}$  values as illustrated in slide 24
- Copy and paste all your MATLAB plots into a single Word document
- To copy a plot, select **Copy Figure** from the **Edit** menu in the figure window (see slide 25)

# Saving Nyquist Plots

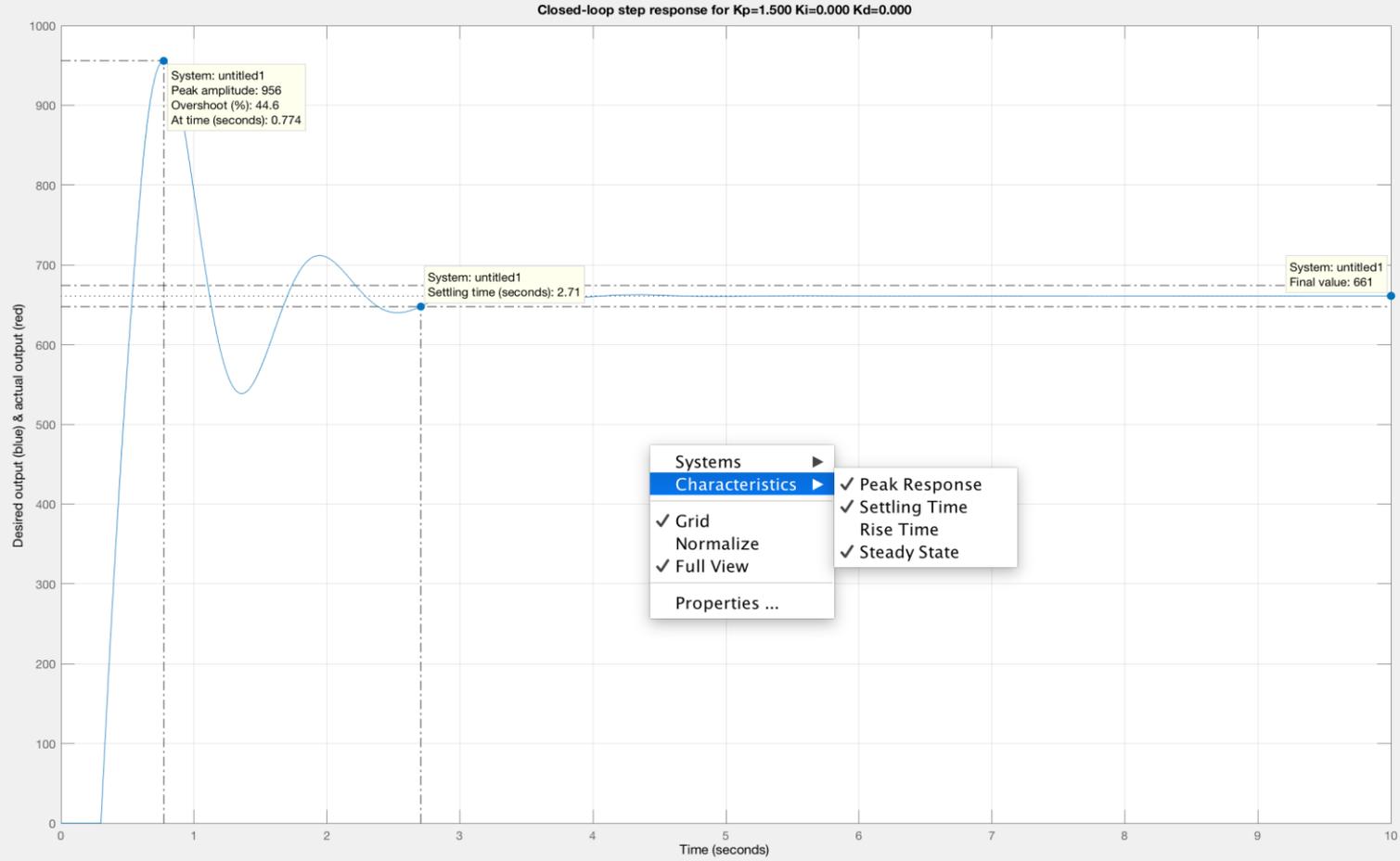


# Saving Bode Plots

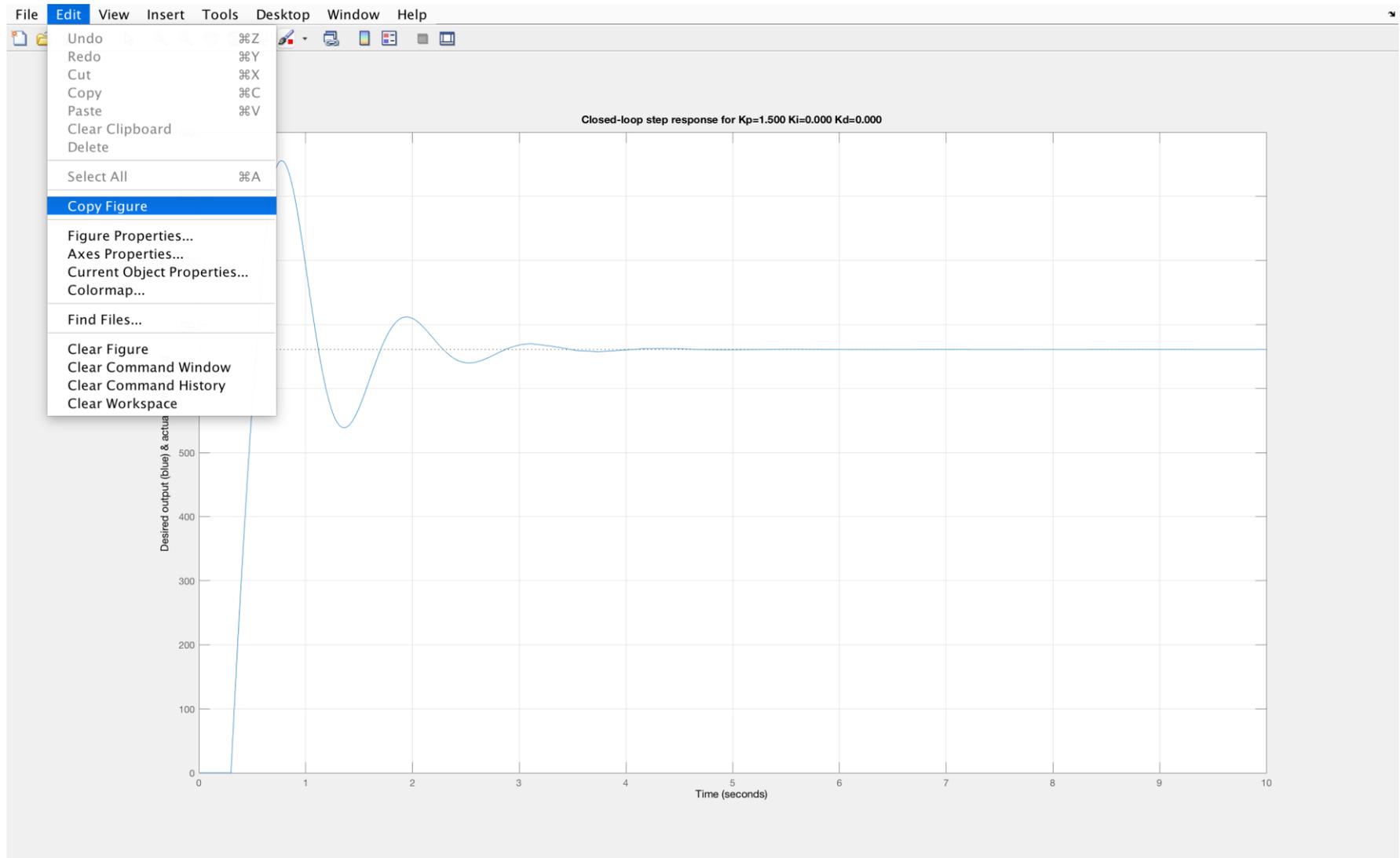


# Saving Step Response Plots

File Edit View Insert Tools Desktop Window Help



# Copying Matlab Plots



## Task 2

- Calculate the Ziegler-Nichols PI controller gains using the formulas

$$M = 1000, \quad K_p = \frac{0.9M}{RT_d}, \quad K_i = \frac{0.3}{T_d}$$

- Calculate the Cohen and Coon PI controller gains using the formulas

$$r = \frac{T_d}{T_p}, \quad K_p = \frac{1}{rK} \left( 0.9 + \frac{r}{12} \right), \quad K_i = \frac{9 + 20r}{T_d(30 + 3r)}$$

## Task 2 (contd)

- In MATLAB, open the simulation programme ***Task2\_L6\_PT\_Lab.m***
- Edit lines 10-12 to assign the values of the process parameters obtained in Task 2
- Run the program.
- Enter the calculated Ziegler-Nichols PI controller gains when prompted
- Enter the calculated Cohen and Coon PI controller gains when prompted

## Task 2 (contd)

- One set of Nyquist, Bode, and step response plots will be produced for the Ziegler-Nichols PI controller. A second set of plots will be produced for the Cohen and Coon PI controller
- Annotate each plot (see slide ), copy the the annotated plot to your Word results document, then close the plot window.

# Task 3 – Nyquist Tuning (contd)

- In MATLAB, open the simulation programme ***Task3\_L6\_PT\_Lab.m***. Edit lines 10-12 to assign the values of the process parameters obtained in Task 2, then run the program.

At the prompt

***‘Enter desired gain margin : ’***

type 2 and hit return.

At the prompt

***‘Enter a trial value of integral gain  $K_i$  : ’***,

type  $0.8/T_p$  and hit return.

# Task 3 – Nyquist Tuning (contd)

- One set of Nyquist, Bode, and step response plots will be produced.
- Annotate each plot (see slide ), copy the the annotated plot to your Word results document, then close the plot window.

# Task 4 Effect of Gain Margin ()

- In MATLAB, open the simulation programme ***Task4\_L6\_PT\_Lab.m***. Edit lines 10-12 to assign the values of the process parameters obtained in Task 2. The program has to run four times with, each time with a different desired gain margin from the set {1, 2, 3, 4}
- At each run of the program you will get the prompt ***'Enter desired gain margin : '***
- Respond to the prompt by typing a value from the above set and hit return.
- Each run will produce a step response plot. Annotate each plot (see slide ), copy the the annotated plot to your Word results document, then close the plot window.

# Task 5 Implement Tuned PI Controllers

## Using the Process Trainer keyboard:

- Switch to closed loop mode
- Enter the values of the Ziegler-Nichols tuned controller gains (from Task 2a), and print the closed loop step response
- Enter the values of the Cohen and Coon tuned controller gains (from Task 2b), and print the closed loop step response
- Enter the values of the Nyquist tuned controller gains (from Task 3), and print the closed loop step response
- Carry out manual fine tuning of the Nyquist tuned controller , and print the closed loop step response

# Report

## Discussion of the Results

Assess the stability, relative stability, and speed of response of each closed loop system, and compare the different closed loop systems, based on:

- The  $GM$  and  $PM$  values from the frequency response plots
- The  $M_p$  and  $t_s$  values from the step response plots

# Report

- **See the separate Assessment Brief document issued to you.**
- **Turnitin submission via Blackboard**
- **Due date: 3 weeks from date of lab**
- **Latest submission time: 4pm**