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Finite Element Analysis of a Vibrating Guitar String and Project Management

Author: Michael Collins

Supervisor: Dr. Conor McCarthy

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I declare that this is my work and that all contributions from other persons have been appropriately identified and acknowledged.

Executive Summary

The aim of this team project was to test the validity of the concept that an automatic tremolo device can be developed for an electric guitar which mechanically alters the string tension using an electronic control system. To achieve this, the project was divided into three roles:

- To design and manufacture a test rig which in conjunction with the group proves the concept of creating an automated tremolo effect on a guitar string

- To design and build an automatic control system for use with the test rig to obtain results to prove the concept above

- To perform a theoretical and finite element analysis of the guitar string for various initial conditions and external excitations. This role also encompassed a project management position

As this is a team project, there is shared data between all three reports. Data that has been gathered is shared in this report and has been referenced accordingly with the permission of the other members involved in the project. This report will focus on the theoretical and finite element analyses that were undertaken, in addition to the project management role in the group.

Abstract

This project aimed to develop an accurate finite element simulation of a vibrating guitar string and use this model to analyse the relationships that existed between the governing parameters of the string. The underlying purpose of the investigation was to verify the design of an experimental test rig which sought to validate the principle concept behind the development of an automated tremolo system for an electric guitar. The project also included a project management role which entailed responsibility for ensuring the project ran cohesively and that regular communication between group members and the group supervisor was maintained. A theoretical analysis was initially performed to provide design specifications for the test rig and help support the findings of the simulation and experimentation processes. The finite element analysis determined the fundamental frequency of the string and its natural overtones in standard tuning, followed by the response of the string as its governing parameters were varied. The deflection profile of a central string node was then analysed for five oscillation cycles of free vibration immediately following a simulated initial excitation. The results of the finite element analysis correlated very closely with those produced in the theoretical analysis, and discrepancies between these results and those obtained from the test rig were attributed to the effects of frictional forces acting on the string at a pivot point in addition to the impact of a bending moment existing in the base plate of the test rig when the string was in tension. It was suggested that a more extensive simulation could encompass the effects of these additional forces and provide further validation of the concept underlying the test rig.

Acknowledgements

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Thanks to my friends and family for their moral support.

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Nomenclature

Symbol	Description	Unit	Symbol	Description	Unit
c	Velocity	m/s	D_n	Constant for n condition	-
f_1	1st Harmonic	Hz	E	Young's Modulus	Pa
f_n	nth Harmonic	Hz	F	Force	N
l	Scale Length	M	F_1	Function 1	-
n	Number	-	F_2	Function 2	-
r	Radius	m	G	Function	-
t	Time	secs	T	Tension	N
x	x-coordinate	m	Y	Function	-
y	Deflection	M	ε_x	Strain in x direction	-
$\dot{y}$	Velocity of Deflection	m/s	θ	Angle	Rad
z	z-coordinate	m	λ	Wavelength	m
A	Area	m^2	μ	Linear Density	kg/m
A	Constant	-	π	Pi	Rad
B	Constant	-	σ_x	Stress in x direction	Pa
C	Constant	-	ω	Angular Velocity	Rad/s
C_n	Constant for n condition	-	ω_n	Angular Velocity for n condition	Rad/s
D	Constant	-	Δl	Extension	m

1. Introduction and Objectives

1.1 Background

Many electric guitars employ a means of rapid pitch variation over a range of frequencies. This is achieved by altering the tension in the strings to create an undulation of the fundamental frequency (natural frequency) at which the strings vibrate (Sundberg 1994). The resulting effect is known as “vibrato”, but is regularly used interchangeably with the term “tremolo”. The aim of this musical technique is to place emphasis on a note and it is employed in various playing styles. For electric guitars, this can be achieved by manually adjusting a lever called a tremolo bar, also known as a “tremolo arm” or “whammy bar”. The fundamental frequencies of the strings are lowered by pushing down on the bar and raised by pulling it up, with rapid operation producing an exaggerated effect (Li, Marcus, Kale, Hoffman, et. al 2006). Various techniques employing electronic manipulation of the frequency output to simulate a tremolo effect exist, dating back to the early 20th century. However, very few commercial applications have involved automating the mechanical process itself of varying the tension present in the strings.

A group project was created with the aim of testing the concept that this manual tremolo technique could be automated, leading to the possible development of a cost effective commercial device that could be attached to or in-built in an electric guitar. The project was divided into three individual roles. These were:

(i) To design and construct a mechanical rig to test the validity of the concept experimentally for a single string.

(ii) To design an electronic control system that could mechanically vary the tension in the string and thus shift its fundamental frequency over a desired range.

(iii) To verify the experimental results obtained from the test rig by carrying out a theoretical and finite element analysis of the vibrating string, and act as project manager for the group.

Role (iii) in the group was the focus of this Final Year Project. The parameters for the test rig and control system were theoretically calculated by employing the wave equation for the special case of vibrating string. These results were used to determine the design specifications for the experimental work (O’ Connor 2011; O’ Neill 2011). The simulation was carried out using Abaqus/CAE finite element analysis (FEA) software, which allowed for extensive characterisation of the physical properties of the guitar string. The fundamental frequency of the string and its natural harmonics were determined for a range of applied tensions, and the effects of varying the governing parameters responsible for the specific fundamental frequency of the string were recorded. By simulating the application of a plucking force to the model, the deflection response of the string was recorded for a magnitude of impulse representing that applied with the experimental rig.

The project management position in the group entailed the responsibility of ensuring that the three roles ran cohesively over the course of the project and integrated smoothly as the work entered the experimental testing phase. A project plan and timeline were drawn up to separate the roles into individual tasks, enabling a more accurate measure of group progress. Regular communication was established between project members through routinely scheduled group meetings. This role also held the responsibility of acting as a point of contact between the group and the project supervisor in order to provide regular updates on progress.

Two tests were performed with the rig which measured the relationship between tension and extension in the string as well as the relationship between tension and fundamental frequency. Data was pooled between group members following the experimentation phase so that correlations could be measured between the theoretical, finite element and experimental analyses.

1.2 Objectives

- To carry out a theoretical analysis of a vibrating string to aid in the definition of the test rig design specifications and help validate the experimental results
- To create a finite element computer simulation of the vibrating string and determine the fundamental frequency, natural harmonics and response of the model for various initial conditions
- To determine the deflection response of the freely vibrating string after it is subjected to an initial excitation
- To perform the role of project manager for the three-man project group

1.3 Chapter Summary

Chapter 1 is this section, which provides the background to the project and defines the objectives of the work undertaken.

Chapter 2 is the literature review. Literature regarding the mathematical and finite element modelling of vibrating systems is discussed. Additionally, the established principles of project management are discussed regarding the attributes of a good manager, the process of defining the scope of a project and planning a project.

Chapter 3 is the theoretical analysis section, in which the major equations used in the report are derived from first principles, and all assumptions made about the vibrating string system are clearly stated.

Chapter 4 outlines the governing parameters of string vibration, as well as the variations in commercial strings that are available. The mathematical analysis of the vibrating string is also discussed, including the process of selecting the correct string for the project.

Chapter 5 is the numerical analysis chapter. This section outlines in full the process of defining and modelling the string using finite element analysis software. The model files and input file are included inside the back cover of the report.

Chapter 6 provides details of the project management work undertaken throughout the duration of the group project, including how the group communicated and interacted.

Chapter 7 is the experimentation chapter which provides details of the experimental apparatus used to analyse the real string. Details of the test procedures are attached in the appendix of the report.

Chapter 8 is the results and discussion section. The results of the finite element and theoretical analyses are compared to the experimental results, and discrepancies in the results are accounted for.

Chapter 9 is the conclusions and recommendations for future work chapter which draws conclusions from the results of the projects and uses this information to suggest possible developments that could be undertaken in future projects.

2. Literature Review

2.1 Mathematical and Computational Modelling of a Vibrating String

A vibrating string is a dynamic system for which the variables are time-dependent. These variables consist of inputs such as excitations and outputs such as the frequency and deflection responses of the string. The response of the string depends upon the initial conditions it is subjected as well as external excitations (impulses). The majority of real vibrating systems are very complex. Thus it is usually very difficult and impractical to compile enough information about the system to formulate a perfect mathematical model. Standard analyses only consider the most important properties of the system in order to make predictions about the response of the string to various specified inputs (Rao 2004).

It is often sufficient to analyse a greatly simplified representation of the vibrating system and still obtain accurate results. Thus the mathematical analysis of the string involves deriving the governing equations for the system based on the specific conditions being tested, solving those equations and interpreting the results (Rao 2004).

The mathematical analysis should take into account whether the vibrating string conforms to a linear or non-linear model. The linear model provides a less complex solution, representing small amplitudes of transverse vibration when the string is excited. This model is based upon the assumption that the string is composed of a linear elastic material such that the restoring force acting on the string when it is displaced from equilibrium is proportional to the amplitude of the deflection. Non-linear analyses are applied to large amplitude deflections and are more complex to solve (Chen and Ding 2007). A standard plucking impulse applied to a regular guitar string results in relatively small amplitudes of deflection, so the linear model can be applied.

A free vibration in a string is one in which no external force is applied to the system after an initial excitation. A string which has been plucked once will result in a free vibration which responds dynamically but not sinusoidally, as no periodic force is applied (Harker 1983).

A good mathematical model should include enough information to describe the string behaviour in terms of equations without becoming too complex. Once the model has been well-defined, the governing equations can be derived by drawing the free body diagram of the mass involved. An element of the string can be taken in isolation if all externally applied forces are indicated. The equations of motion are in the form of a set of partial differentiation equations for the continuous string model. Solving these governing equations produces the vibrational response of the string (Rao 2004).

Numerical methods can also be performed by computers to solve the governing equations. The finite element analysis technique employs a method of mesh discretisation of continuous systems into a system of discrete sub-domains called “elements”. Thus both the dependent and independent variables are treated as discrete quantities to make them suitable for digital computation, creating a discrete-time system (Meirovitch 1986).

2.2 Established Project Management Techniques

2.2.1 Choosing a Project Manager

Much work has been published on principles of project management spanning the range of characteristics that a good project manager should possess as well as the responsibilities he/she is required to undertake in order to effectively manage workflow within a group. When evaluating a candidate’s suitability for this role, it is necessary to gauge whether the background, skills and experience of the person are sufficient to fill an engineering project management position. According to Ludwig (1988), some of the major factors to consider in this evaluation are:

(i) Leadership skills
(ii) Personality
(iii) Relevant experience
(iv) Patience
(v) Maturity of judgement
(vi) Process Understanding
(vii) Breadth of interest

An ideal candidate should display strengths that pertain to each of these areas and be able to effectively balance his or her skillset as appropriate for the job at hand. The project manager has the responsibility of ensuring that tasks are completed in a timely and satisfactory manner within budget, but does not always have formal authority over those actually carrying out the work. He or she must therefore rely on techniques of persuasion and negotiation in order to effectively influence the various parties falling within the scope of the project work (Shtub et. al 1994).

2.2.2 Project Scope

Outlining the project scope involves defining and controlling what work does or does not fall within the boundaries of the project tasks. The size, complexity and importance of the project in addition to several other factors determine how much effort is required in planning the scope of the project. A well-defined scope is crucial to the success of a project because it helps to improve the accuracy of time, cost and resource estimates, illustrated in Fig. 2.1. A scope statement is often prepared in industry to briefly and concisely describe the focus and range of the project (Schwalbe 2009).

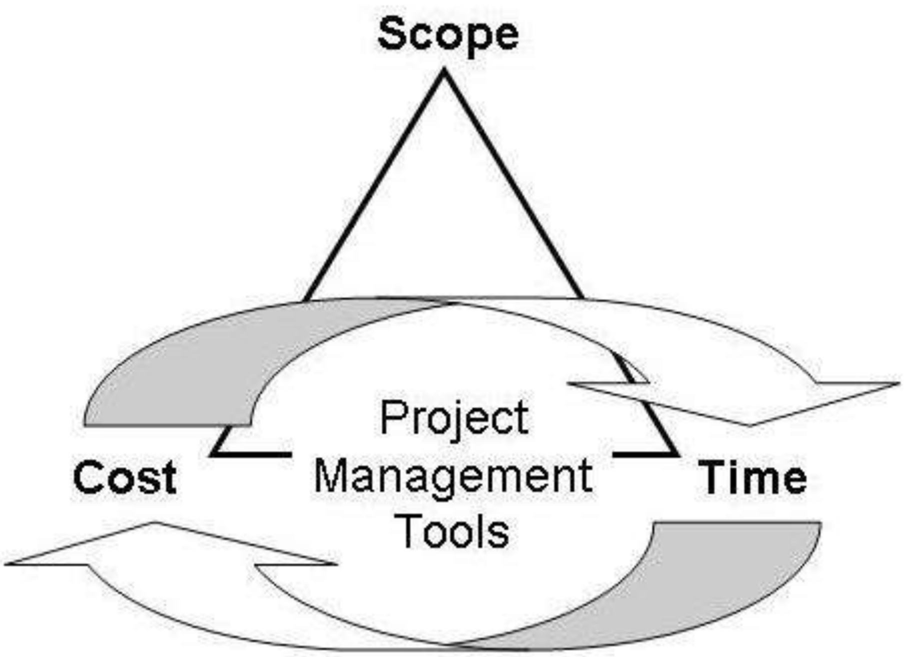

Figure 2.1 – Pyramid relationship between project scope, cost, time and management tools (Price Systems (2008).

2.2.3 Project Planning

Once the scope of the project has been finalised, the project proceeds to the planning phase. According to Howes (2001), this phase can be subdivided into five activities, consisting of:

(i) Subdivision of work

The work is divided into parts and sub-parts by devising a work breakdown structure (WBS), as seen in Fig. 2.2. This process enables the project manager to balance tasks effectively between team members, and provides a concise overview of progress.

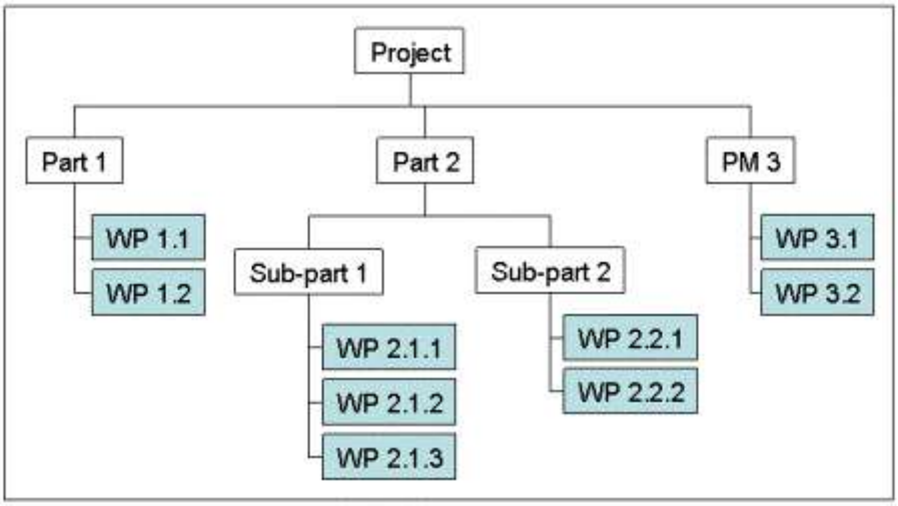

Figure 2.2 – Sample work breakdown structure (Project Management Knowhow 2011).

(ii) Quantification

Quantification is the process of assigning a "unit of measure" to each task or sub-task. For example, a task involving the writing of computer code may be quantified in terms of "lines of code", or a task involving digging a hole may be quantified in terms of "cubic feet" that need to be dug. This technique offers a means of gauging progress within individual tasks.

(iii) Sequencing of work

Sequencing refers to the arranging of these tasks within the project, and is often determined using a precedence diagram. These diagrams display the dependency relationships between tasks. For instance, in order to begin task 2 it may be a requirement to perform task 1 first. Therefore, tasks 1 and 2 are said to have an "end-to-start" dependency. There may also be "start-to-end", "start-to-start" and "end-to-end" dependencies. Fig. 2.3 illustrates this concept.

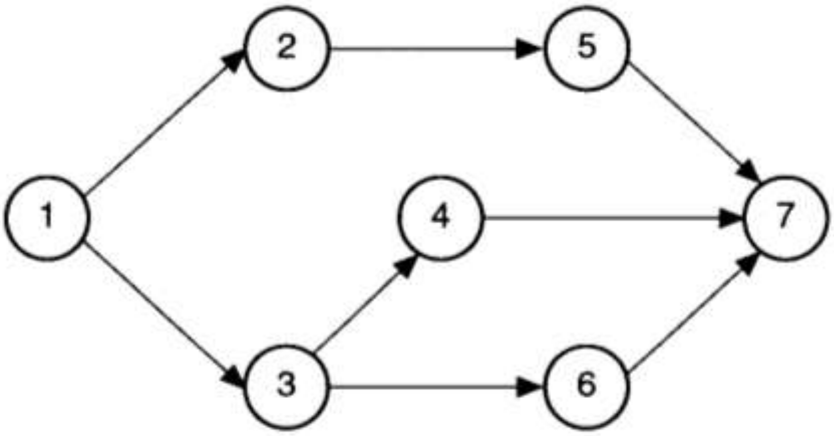

Figure 2.3 – Typical Precedence Diagram (Emerald Insight (2011).

(iv) Budgeting

Budgeting for the project is achieved by estimating the hours of labour and cost required to perform each task. The cost and labour hours required for each task are estimated using the quantification for that task. The sum of the costs and labour hours for each task combined with the cost of resources provides an estimate of the total cost of the project. This information is used to outline an initial budget model, which may require adjusting over the course of the work to account for profits and deficits made on each individual task.

(v) Scheduling

After estimating the costs, labour hours and resources required to carry out each task, they can be applied to the sequence of tasks outlined in the precedence diagram, creating the project schedule. Scheduling also involves setting start and end time estimates for each task so that predictions and deadlines can be set for the project.

3. Theoretical Analysis

The guitar string is assumed to be a flexible, homogenous, isotropic body which adheres to Hooke's Law within its elastic limit. In order to determine the position of every particle in the continuous system when it undergoes vibration, an infinite number of coordinates are required. As such, the string possesses an infinite number of degrees of freedom at all positions along its length excluding the fixed end points.

When the string is subjected to an excitation it undergoes a free vibration that is the sum of the principle modes of the string. For the principle mode of vibration, every particle of the string undergoes simple harmonic motion at the frequency corresponding to the root of the frequency equation (see Eq. 3.14) at that particular instant. In other words, each particle passes through its respective equilibrium position simultaneously.

A string of linear density (mass per unit density) μ is subjected to a tension T. The vertical deflection y of the excited string is assumed to be very small, producing a negligible change in tension which can be ignored for this analysis. Vibrations in the string are assumed to be linear, so that the maximum deflection achieved by the string in vibration remains proportional to the elastic restoring force acting upon it.

Taking an arbitrary element dx of the string on its own as it is moving in the y-direction due to a free vibration, the free body diagram for that element can be produced, shown in Fig. 3.1.

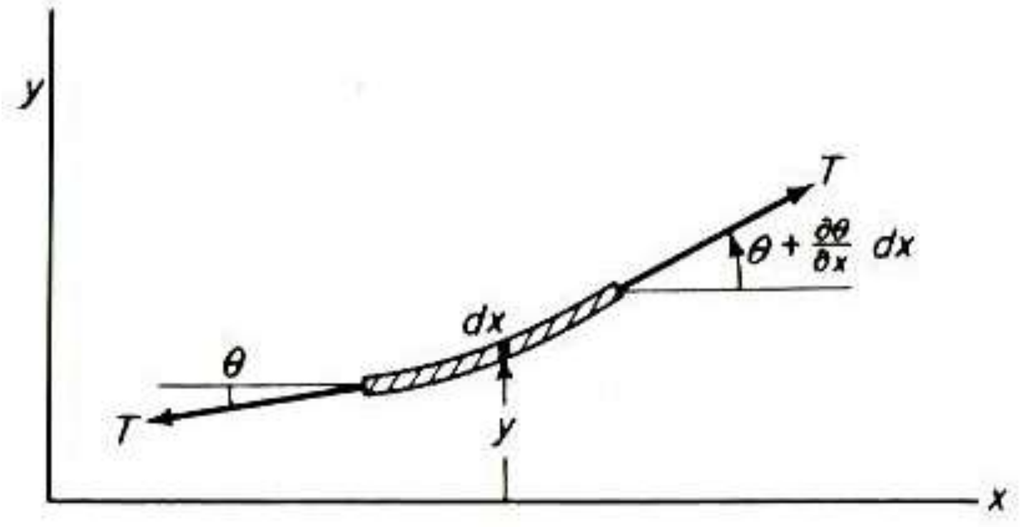

Figure 3.1 – String element undergoing vibration in the y direction (Thomson 1986).

The equation of motion for this element in the *y*-direction can be expressed as

$$T\left(\theta + \frac{\partial\theta}{\partial x}\partial x\right) - T\theta = \mu dx \frac{\partial^2 y}{\partial t^2} \tag{3.1}$$

This can be simplified to give

$$\frac{\partial\theta}{\partial x} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} \tag{3.2}$$

From Fig. 3.1 it can be seen that the slope of the string element is $\theta = \partial y/\partial x$, which when substituted into Eq. 3.2 produces

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} \tag{3.3}$$

where $c = \sqrt{T/\mu}$ is the velocity of the wave as it propagates through the string.

Taking the generalised form of Eq. 3.3 and expressing it in terms of the deflection *y* gives

$$y = F_1(ct - x) + F_2(ct + x) \tag{3.4}$$

where F_1 and F_2 are unknown functions. For any type of function that F may be, differentiation of the argument $(ct \pm x)$ produce

$$\frac{\partial^2 F}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \tag{3.5}$$

which satisfies the differential equation.

The values of the components of Eq. 3.4, $y = F_1(ct - x)$ and $y = F_1(ct + x)$, are determined by the values of their respective subcomponents, $(ct - x)$ and $(ct + x)$. Therefore, they are determined by a range of values of x and t for any fixed wave propagation velocity *c*. In simpler terms, a wave of constant velocity *c* passes any point

x on the string at time t. The argument $(ct - x)$ represents a wave travelling in the positive x-direction, whereas the argument $(ct + x)$ represents a wave travelling in the negative x-direction.

Using the separation of variables technique to solve the partial differential equation, the solution takes the form

$$y(x,t) = Y(x)G(t) \tag{3.5}$$

Substituting this into Eq. 3.5 gives

$$\frac{1}{Y}\frac{d^2Y}{dx^2} = \frac{1}{c^2}\frac{1}{G}\frac{d^2G}{dt^2} \tag{3.6}$$

The left side of the resulting equation is independent of t and the right side is independent of x. This infers that both sides of the equation must be a constant. Putting this constant equal to $-(\omega/c)^2$ produces the differential equations

$$\frac{d^2Y}{dx^2} + \left(\frac{\omega}{c}\right)^2 Y = 0 \tag{3.7}$$

$$\frac{d^2G}{dt^2} + \omega^2 G = 0 \tag{3.8}$$

which have the general solutions

$$Y = A\sin\frac{\omega}{c}x + B\cos\frac{\omega}{c}x \tag{3.9}$$

$$G = C\sin\omega t + D\cos\omega t \tag{3.10}$$

The resulting constants A, B, C and D in this equation are dependent upon the boundary conditions and the initial conditions for the string. For this analysis the string is in tension, stretched between two fixed end points. If the distance between these end points is l, the boundary conditions will be $y(0,t) = y(l,t) = 0$. In order for the boundary condition $y(0,t) = 0$ to be true (no vertical deflection at the point $x = 0$), it requires that the condition $B = 0$ is also true. This solution yields

$$y = (C \sin \omega t + D \cos \omega t) \sin \frac{\omega}{c} x \tag{3.11}$$

The boundary condition $y(l,t) = 0$ then produces the equation

$$\sin \frac{\omega l}{c} = 0 \tag{3.12}$$

or

$$\frac{\omega_n l}{c} = \frac{2\pi l}{\lambda} = n\pi, \quad n = 1,2,3 \ldots \tag{3.13}$$

where $\lambda = c/f$ is equal to the wavelength and f is the frequency of the vibration. Every value of n represents a different mode of vibration for the oscillation, from its fundamental mode through an infinite number of natural overtones determined by the harmonic series corresponding to the fundamental. The particular frequency of the mode is determined from

$$f_n = \frac{n}{2l} c = \frac{n}{2l} \sqrt{\frac{T}{\mu}}, \quad n = 1,2,3 \ldots \tag{3.14}$$

where the mode shape provides a sinusoidal distribution determined by the equation

$$Y = \sin n\pi \frac{x}{l} \tag{3.15}$$

For the fundamental frequency of the string, the solution therefore takes the form

$$f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \tag{3.16}$$

Subsequent overtones of the fundamental were can be calculated for increasing values of n.

The general form of Eq. 3.15 used to analyse free vibrations initiated by any method of excitation accounts for all modes of vibration propagated in the oscillation, and can be written as

$$y(x,t) = \sum_{n=1}^{\infty}(C_n \sin \omega_n t + D_n \cos \omega_n t) \sin \frac{n\pi x}{l} \tag{3.17}$$

where

$$\omega_{n=} \frac{n\pi c}{l} \tag{3.18}$$

Eq. 3.17 is used to determine the amplitude of the free vibration upon excitation, using the initial conditions of $y(x,0)$ and $\dot{y}(x,0)$ to determine the values of C_n and D_n (Thomson 1986).

4. String Properties and Theoretical Work

4.1 Governing Parameters

Guitar strings are manufactured based on three major governing parameters which determine the fundamental frequency that the string will vibrate at, these being:

(i) Scale Length
(ii) Tension
(iii)Linear Density (Mass per Unit Density)

Different types of strings vary with respect to these parameters, with each type producing a unique harmonic pattern of vibration. This provides a range of specific sets of musical characteristics that can be tailored to suit the individual player or playing technique.

(i) Scale Length

The length of the string is an important factor which determines its fundamental frequency. The scale length of the string refers to its "working length", which is the length of string that is actually oscillating when plucked. The scale length is determined by the distance between the bridge and nut of the guitar, and can vary for different guitar models. The nut and bridge are fixed points which are in contact with the string when it is in tension, limiting the section of string that is free to vibrate. Fig. 4.1 illustrates this concept, showing how the string is free to vibrate between two fixed end points that determine the magnitude of the scale length.

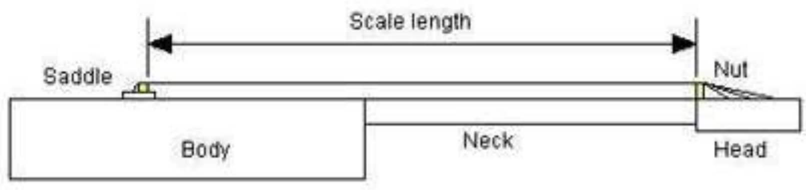

Figure 4.1 – Guitar Scale Length (How Stuff Works (2001).

The scale length has an inversely proportional relationship with fundamental frequency. Therefore, when the scale length is reduced to half its original length the frequency is doubled, and when the scale length is reduced to a quarter of its original length the frequency is quadrupled, etc. This relationship is expressed in Eq. 4.1.

$$l \propto \frac{1}{f_1} \tag{4.1}$$

Frets are devised based upon this relationship between scale length and frequency. Frets are a series of metallic contact points that lie just below the strings on a guitar along the fret board. When the string is pressed down so that it makes contact with a fret, the scale length is reduced so that the string can only oscillate between that fret and the bridge, producing a higher frequency note. Using the mathematical relationship between these properties, a fret board can be calibrated with frets such that any desired note can be played. In order to reach higher and higher frequencies, the scale length is first divided in half, then into thirds, quarters, fifths and so on. As a result, the scale length will be shortened in progressively smaller increments as the frequency increases. Therefore, the distances between each fret (the fret spacings) are seen to grow smaller and smaller as the note increases in frequency, as seen in Fig. 4.2.

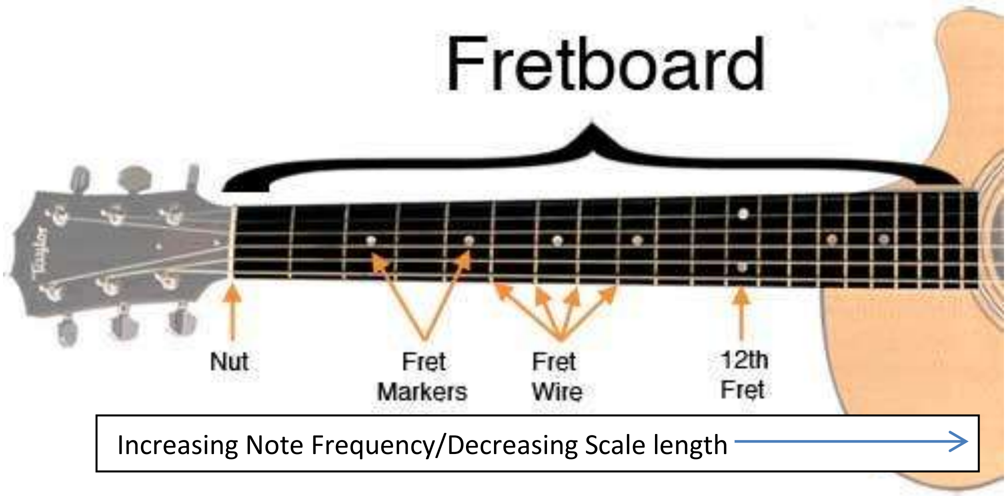

Figure 4.2 – Standard fret board (Start Playing Guitar 2011).

(ii) <u>Tension</u>

The next important parameter that affects the fundamental frequency of the string is the amount of tension it is being subjected to at a given instant. When a tensile stress is applied to a string along its longitudinal axis the string elastically deforms and undergoes longitudinal strain, becoming increasingly taut as the magnitude of the extension increases. This increase in potential energy in the taut string allows it to oscillate at higher frequencies when it is disturbed by an impulse, such as when it is plucked by a guitar plectrum. Thus it follows from Eq. 4.2 that there is a directly proportional relationship between tension and fundamental frequency.

$$T \propto f_1 \tag{4.2}$$

(iii) <u>Linear Density</u>

The linear density also has a significant impact on the fundamental frequency of the string in vibration. Linear density is a measure of the mass per unit length of the string. The greater the liner density of a string is, the slower it will oscillate due to increased inertia in conjunction with increased internal and external frictional forces acting upon the mass. Therefore, heavier strings will vibrate at lower frequencies than lighter ones, as described by the inverse relationship in Eq. 4.3.

$$\mu \propto \frac{1}{f_1} \tag{4.3}$$

4.2 Variations in Commercial Strings

4.2.1 Overview

Guitar strings come in a variety of forms dependent upon the desired playing style of the guitarist. A standard string consists of a central core wire of circular or hexagonal cross-section which is then either plated or wound (wrapped) in another material. The thicker gauge strings (E_2, A_2 and D_3) on the guitar are always wound, whereas the thinner strings (B_3 and E_4) are left “plain” (unwound). However, the G_3 string can be either plain or wound, typically being left plain on an electric guitar but often being wound for acoustic string sets.

The variables that determine the physical parameters of the string (mentioned in section 4.1) and thus the tone of the sound produced by its vibration are:

For plain strings

(i) The material used to construct the string
(ii) The material the string is plated in

For wound strings

(i) The material used to construct the core wire
(ii) The material used to construct the winding
(iii) The diameter of the core relative to the winding
(iv) The technique used to wrap the core

A guitar string which receives an impulse will simultaneously oscillate with a specific set of frequencies. This pattern of vibration is composed of the strings fundamental frequency, which is the loudest and most audible frequency, in addition to a complex pattern of natural overtones of the fundamental frequency. The specific harmonic pattern (tone) produced by the vibration is directly influenced by each of the variables listed above. Therefore, variations in these properties will result in a string that vibrates in different combinations of its harmonics, creating a different quality of note.

4.2.2 Plain Strings

Plain strings are typically manufactured from steel, nylon or fluorocarbon. Plain steel strings are ideal for electric guitars as they have strong magnetic properties which are a requirement in order for the string to work in conjunction with the pickups by means of electromagnetic induction. They are typically plated with tin or brass to prevent oxidation and the subsequent build-up of rust on the string over the duration of its life time, as rust can result in a reduction in the quality of the note produced by the string (Stringbusters 2005).

4.2.3 Wound Strings

Thicker gauge strings are wound in order to increase their mass and produce a lower frequency without compromising in string flexibility. They are typically wound in a bronze, nickel or steel material. Electric guitar strings are required to exhibit magnetic properties in order for electromagnetic induction to take place between the oscillating string and the row of coiled bar magnets that form the pick-ups fitted in the body of the guitar. Therefore, in the case of steel windings, an inferior grade of stainless steel must be used for their magnetic properties as pure stainless steel is non-magnetic (Stringbusters 2005).

Strings can be wound using three different main techniques which have a bearing on the tone produced by the string. These are roundwound, flatwound and halfwound, displayed in Fig. 4.3.

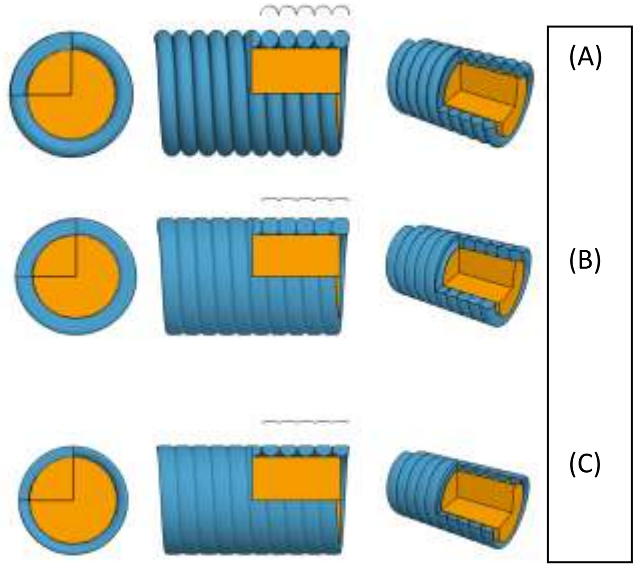

Figure 4.3 – Three main winding techniques: (A) roundwound, (B) flatwound and (C) halfwound (The Bass Guitar Website 2011).

Roundwound wire has a circular cross section, and produces a “brighter” tone than the other techniques. However, it is susceptible to a lot of undesirable finger scraping noise as the hand slides up and down the string due to its bumpy surface. Flatround wire has a more square-shaped profile, resulting in a much smoother string surface that produces less frictional noise as roundwound. These strings produce a more “mellow” tone. Halfwound wire has the cross section of a roundwound wire that is cut in half, flattened on the playing side. This produces a “bright” sound without as much scraping noise as roundwound strings (The Bass Guitar Website 2011).

4.2.4 String Gauge

Strings are manufactured in a variety of different diameters or “gauges”, increasing in gauge from the 1st string (E_4) to the 6th string (E_2). Guitars are played using strings of increasing gauge in order to allow the player access to a wide range of musical notes across numerous octaves on the fret board. A six string guitar with a fret board consisting of 24 frets provides access to four full octaves.

A standard set of six guitar strings will vary in gauge from around 0.23mm (0.009 inches) for the thinnest string to around 1.17mm (0.046 inches) for the thickest string, with some variation in this regard for different manufacturers and playing styles. Sets of strings are generally gauged so that the strings feel roughly equal in tension to the player. When standardised tension values are applied to the six strings, the guitar is said to be in “standard tuning”. This means that when plucked, each individual string will vibrate at its own standardised fundamental frequency. This concept is illustrated in Fig. 4.4, which displays a set of strings tuned up on a guitar from 1st to 6th and the musical notation associated with the fundamental frequency of each string.

Figure 4.4 – Strings in standard tuning (Guitar Savvy 2010)

4.3 Theoretical Work

The string being modelled in the mathematical analysis was a D'Addario plain steel G_3 string which is typically used on both electric and acoustic guitars (D'Addario 2011). The G_3 string was chosen as it was the thickest gauge string that was not wound in another material. Working with a plain string greatly simplified the calculations, as it was not necessary to take into account the non-uniform profile and inconsistent mechanical properties of a wound string composed of a combination of different materials. Choosing the thickest gauge plain string lowered the risk of accidentally exceeding the yield strength of the string during experimental testing, as thicker gauges allowed for more tension to be applied to the string with lower corresponding degrees of stress and strain.

Through communication with D'Addario customer service staff, the exact grade of steel used to produce the string was established in addition to its mechanical properties. The string was manufactured from A228 high grade carbon steel, which has 0.7 - 1.0% carbon content (Kent 1950) and is used to produce high quality musical wire. Using Eq. 3.14, it was possible to determine the range of tension values required to raise and lower the fundamental frequency of the string over the desired range of semitones to create a tremolo effect. This data was used to determine the model of actuator required for the electronic control system, as well as the loading conditions for the test rig (O' Connor 2011, O' Neill 2011).

The effects of varying the linear density, tension and fundamental frequency of the string were also recorded theoretically, and the subsequent relationships were plotted against the finite element simulation results and experimental results, aiding in the validation of the test rig.

5. Numerical Modelling

5.1 Simulation Technique

A D'Addario plain steel guitar G_3 string was modelled to fit the specifications of a standard electric guitar, using the same properties as the string analysed in the mathematical analysis (see section 4.3). This modelling was carried out using Abaqus/CAE version 6.9-1 finite element analysis software created by the Simulia company (Simulia 2011). Abaqus allows for quick and efficient modelling, visualisation, meshing and analysis of various geometries. The software uses arbitrary units of measurement for the convenience of the user. As a result, all units of measurement were converted to S.I. units (International System of Units) prior to the analysis in order to ensure the consistency of the results throughout the analysis. Using the Abaqus technique, the following were achieved:

(i) Created a simulated virtual G_3 string with accurate geometrical and material properties
(ii) Applied tension to the string
(iii) Calculated the fundamental frequency and natural overtones of the string at fixed magnitudes of tension
(iv) Plotted the relationships between tension, frequency and mass per unit length
(v) Measured the deflection of the string as it was subjected to a central plucking force and analysed the resulting harmonic pattern of vibration

5.2 Step 1 – Initial Step

5.2.1 Part Geometry

As the magnitude of the strings length was so much greater than its diameter, it was convenient to model the string as a three dimensional deformable wire. The scale length of the string was sketched as 0.648m, as this is the scale length between the nut and bridge of a standard Fender Stratocaster. Although the real string is required to be much longer than its scale length in order to coil around the machine head to allow for manual tuning, it was only necessary to model the working length of the string for the finite element analysis procedure as this was the only section that was involved in vibration.

A cross sectional profile was created in order to apply the correct geometry to the virtual string model. This was a circular profile of radius 0.215×10^{-3} m which reflected the gauge of 0.43mm (0.017 inches) of the real string. The cross sectional area of the string at this gauge was 1.45×10^{-7} m^2, and its volume for this scale length was 9.41×10^{-8} m^3.

5.2.2 Material Properties

The real string was manufactured by the D'Addario company from round steel. It was a plain steel string that was not wound in another material. The string had a Young's modulu of 210.29GPa, a Poisson's ratio of 0.313 and a density of 7850kg/m^3. This density resulted in a linear density of 1.14×10^{-3} kg/m.

A new material was created in Abaqus that was given these properties, and it was applied to the model geometry. This ensured that the model deformed elastically in response to an applied stress in the same way as the real string did, and that the model would also vibrate at the correct fundamental frequency when subjected to the same amount of tension as the real string.

5.2.3 Mesh

The string was partitioned into 4 equal sections of length 0.162m. This allowed for more extensive analysis at the partition boundary nodes, so that impulses could be applied at points across the string to simulate the harmonic pattern of vibration as the point of excitation is varied.

The 4 partitioned sections were seeded with 200 seeds each, resulting in a total of 800 seeds across the string, which was optimal discretisation for the problem. The number of seeds determined the number of elements in the mesh. Standard, linear order beam elements were chosen for the mesh. Linear order elements were favoured over quadratic elements because the mass of the part would not have been distributed evenly at each node for analyses involving wave propagation for quadratic elements. The seeding produced a mesh comprising of 801 elements, and 801 nodes. There were no mid-side nodes as the mesh was of linear order.

5.3 Step 2 – Applying Tension

After the initial conditions were designated in step 1, a second step was created in order to apply tension to the string. Boundary conditions were set up at both end nodes of the model in order to simulate conditions at the bridge and nut of the guitar, representing the end points of the scale length of the string. At the bridge node, displacements in the x, y and z directions were constrained, as well as rotation about the x axis (longitudinal axis). This reduced the node from 6 degrees of freedom to 2. At the nut node, displacements in the y and z direction were constrained in addition to rotations about the x axis. This allowed for extension of the model in the x direction when tension was applied. The nut node thus had 3 degrees of freedom. Both nodes were free to rotate about the y and z axes. Rotation about the longitudinal axis was damped out for these nodes to simplify the analysis, as a negligible amount of rotation occurred about this axis in the real string.

It was possible to achieve tension using two equally valid methods. The first method was to apply a concentrated tensile load in the x-direction along the longitudinal axis of the string at the nut end node. The second technique was to apply a nodal displacement to the nut node precisely corresponding to the desired tension. Fig. 5.1 shows the tension applied to the nut node of the model.

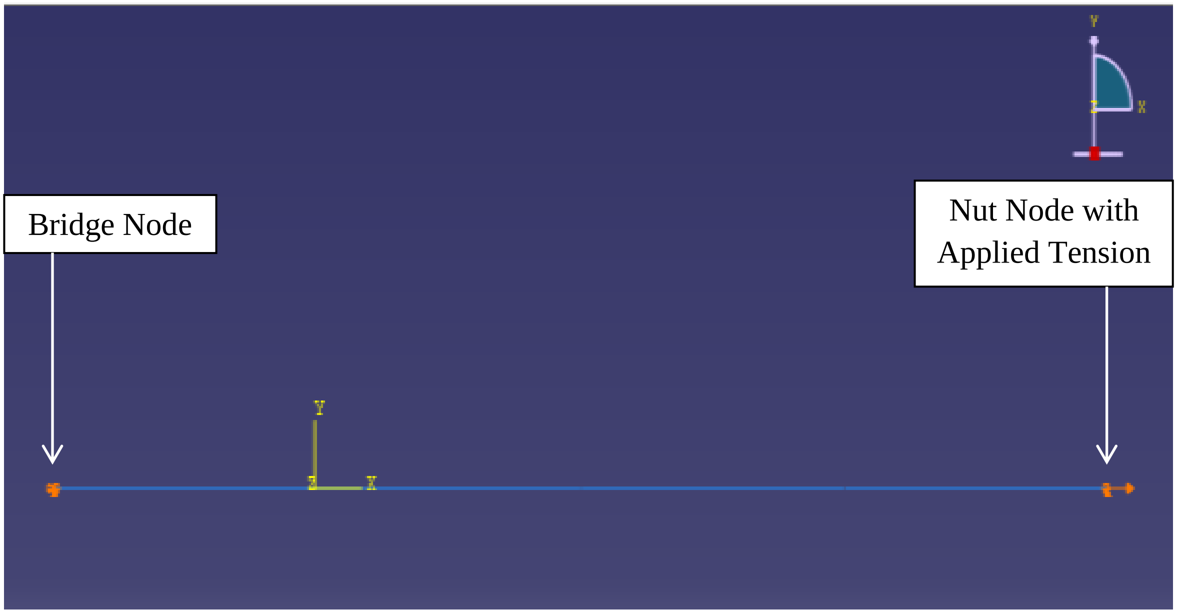

Figure 5.1 – Tension applied to nut node.

This displacement was calculated by employing a number of theoretical relationships. First, the tensile stress δ_x acting on the string in the x-direction was determined using the equation

$$\delta_x = \frac{F}{A} \tag{5.1}$$

where F was the force applied to the nut node and $A = \pi r^2$ corresponded to the cross-sectional area of the string. Next, the resulting strain ε_x in the string due to the tensile stress in the x-direction was calculated from

$$\varepsilon_x = \frac{\delta_x}{E} \tag{5.2}$$

where E is the Young's modulus of the string material. Finally, the extension Δl in the string (i.e. its change in length) was determined from the equation

$$\Delta l = \varepsilon_x l \tag{5.3}$$

Inputting a nodal displacement at the nut instead of a tensile force made it more convenient to monitor and record changes in the scale length of the string and thus interpret the results more accurately with regards to this extension. The change in scale length affected the magnitude of the natural frequency, but this effect was very small due to the tiny amount of strain taking place. Therefore, the extension had a small but measurable impact in the results.

5.4 Step 3 – Modal Analysis

Once the tension was applied, the next step could be created which calculated the fundamental frequency of the string at a particular tension, as well as the sequence of its natural overtones. For this analysis, the string was required to have both the nut and bridge nodes constrained to prevent displacements in the x, y and z directions, as well as rotations about the x axis. A new boundary condition was therefore created at the nut in the modal analysis step to account for the new constraint in the x-direction at this position. The boundary condition at the bridge was maintained from the previous step as it remained unchanged.

As the tension application step involved applying a displacement to the nut node, inducing strain in the string and shifting the position of the nut, it was necessary to specify that Abaqus should carry on the effects of any deformations or displacements resulting from step 2 on to the modal analysis step. This ensured that the new boundary condition was not applied until after the nut node had been displaced to its new position on the x-axis, and also that Abaqus factored the tension into its calculations for the fundamental frequency of the string.

The number of eigenvalues requested determined the number of normal vibrational modes returned from the analysis. These modes were free to vibrate in the x-y and x-z planes for each harmonic.

5.5 Step 4 – Modal Dynamics

The final step in the procedure was the modal dynamics analysis. This step simulated the response of the string when it underwent a free vibration after an initial impulse (excitation) over the course of one second. An instantaneous concentrated force was applied to a single node to initiate the response. The analysis was run three times, varying the node at which the impulse was applied to each time in order to record differences in the patterns of modes which the oscillations went through during a full cycle. These nodes existed at $l =$ 0.162m, $l =$ 0.324m and $l =$ 0.486m. The force was applied instantaneously so that the precise number of oscillation cycles could be accurately visualised over the range $0 \leq t \leq 1$ seconds, such that the condition $t = 0$ coincided exactly with the instant when the node reached maximum deflection in the y-direction due to the impulse. This instant marked the beginning of the first cycle of the free vibration after the source of initial excitation was removed.

The modal dynamics analysis provided a means of visual confirmation of the fundamental frequency of the string. For example, when measuring the fundamental frequency of the G_3 string in standard tuning, the analysis was run with very small time increments (3920 increments per second). This resulted in a frame rate of 20 frames per cycle, allowing the user to clearly visualise a full oscillation and validate that the string goes through 196 cycles per second, confirming the modal analysis result of 196Hz for the string. Additionally, this analysis provided a means of accurately recording the deflection of any node on the string during a free vibration.

5.6 Finite Element Analysis Procedure

The full Abaqus analysis procedure is included in Appendix A of this report.

6. Project Management

6.1 Overview

The group project was divided into three roles, necessitating the appointment of a team member to the position of project manager. This role presented three primary responsibilities:

(i) To oversee the progress of the group project and ensure that all three aspects ran cohesively and on schedule

(ii) To ensure regular communication between team members and hold weekly internal group update meetings

(iii) To act as a point of contact between the group and the project supervisor throughout the academic term, arranging regular meetings to provide updates on group progress to date

6.2 Project Planning

On a monthly basis throughout the project, brief and concise schedules were drawn up for the three group members that listed the tasks requiring completion over the following weeks. The tasks were arranged in order of precedence, and suggested deadlines were proposed and submitted to the other group members for review. Initial feedback from the members resulted in finer adjustments to the schedules based on new information, and the plans were subsequently finalised. This method of monthly scheduling set multiple individual targets for each team member to work towards in the short term as well as over the long term. Breaking the tasks down into their component parts provided a means of predicting their duration and quantifying the work involved in a way that was manageable and measurable.

The following is a sample schedule written up in January and submitted to the group:

(i) 10th - 14th January (current week)

- **John -** Conclude research and decide upon final design to be employed for impulse mechanism
- **Robert -** Conclude research and decision process for load cell

- **Michael -** Correlate fundamental frequency calculations with FEA simulated values
- Arrange meeting with supervisor (Thursday 13th January) to verify final decisions for what impulse mechanism to build, and discuss where we are sourcing the load cell from

(ii) **Friday 14th January**

- **John -** Begin material sourcing and construction of string impulse mechanism
- **Robert -** place order for load cell
- **Michael -** Perform modal analysis for increasing tension

(iii) Friday 21st January

- **John –** Meeting with workshop technicians to learn current status of test rig components

(iii) Semester 2 Week 1, 24th - 28th January

- **All –** preliminary experimental procedures to be drawn up
- **Robert –** Obtain LabView, begin tutorials on how to use the software
- **Michael –** Perform modal analysis for increasing linear density

(iv) Semester 2 Week 2, 31st January – 4th February

- **All –** Begin writing up of FYP reports
- **John –** Write up section on design concepts
- **Robert –** Write up section on various actuator models
- **Michael –** Write up numerical modelling procedure to date

6.3 Group Meetings

Regular contact was established between group members through email and texting. On a weekly basis, 15 - 30 minute meetings were arranged to provide each member with an overview of group progress, as well as a description of individual work performed by team members. In order to take value from these meetings, details were tracked and recorded to be available for later review and a plan of action based on the content of the meeting was established. The following are the minutes recorded from a typical group meeting:

Group Update – 25th January 2011

- Discussed material selection for the base plate of the rig
- Parts must be designed to house the humbucker under the string in the rig
- Received book on theory and finite volume method for a guitar
- Robert completed LabView tutorials
- Need to conclude on correct boundary conditions for the finite element model
- Report structure

<u>Plan of action</u>

- Arrange meeting with supervisor to present preliminary report layout and discuss material for base plate
- Consult textbook for information on boundary conditions used in finite element model

7. Experimentation

7.1 Experimental Apparatus

7.1.1 Overview

The experimental test rig (see Fig. 7.1) was designed and constructed to recreate the conditions acting on the G string when tuned up on a standard electric guitar (O' Connor 2011, O' Neill 2011).

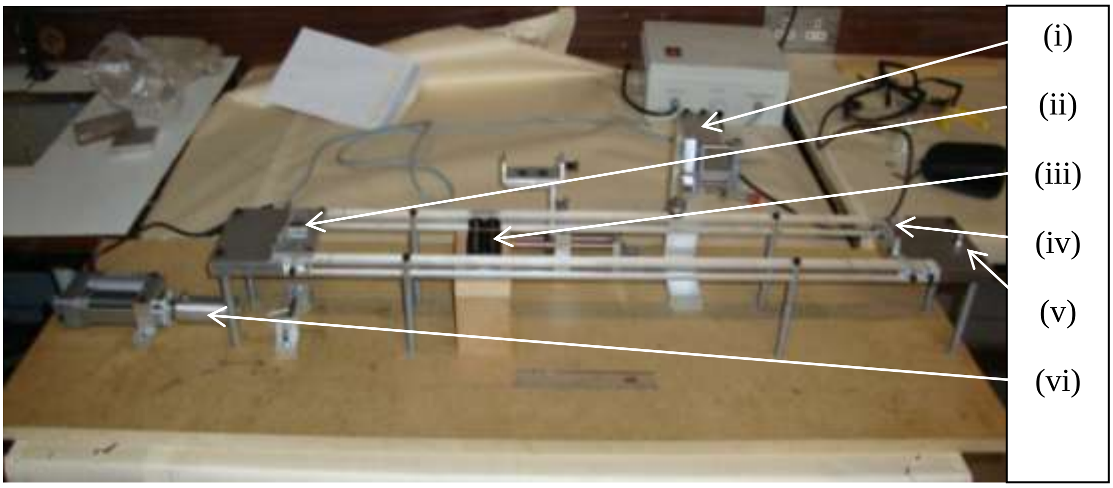

Figure 7.1 – Experimental test rig.

The string was connected to a locking tuner (v) at one end and passed through a clamp (iv) representing the nut of the guitar. The string was then stretched across a distance of 0.648m (the scale length of a standard electric guitar) to a point representing the bridge of the guitar (ii). The excess length of string reaching beyond the scale length was connected to a load cell, which was subsequently screwed onto an actuator (vi).

The rig design enabled precise adjustments to the tension acting upon the string. This was controlled electronically by inputting commands on a laptop which drove the displacement of the actuator connected to the string. This displacement was read manually using electronic vernier callipers. Excitation of the string was initiated by a mechanical pendulum device (i), which simulated the impulse acting upon the string when it was plucked. The tension in the string was read from a tensometer connected to the load cell. The corresponding output frequency for the string undergoing vibration was read by an electronic tuner. A "humbucker" (iii) read and output the frequency of the note produced by the string so that the volume of the sound could be amplified through a loudspeaker, as it is for an electric guitar.

7.1.2 Support Structure

The rig was mounted on a base plate fabricated out of medium-density fibre board (MDF), chosen for its good machinability and lightweight material properties. The nut and bridge points were mounted on two elevated end plates made of aluminium 602A connected across the scale length by two polycarbonate beam sections, both materials selected for their lightweight, non-magnetic properties (non-magnetic to prevent interference with the string vibration and to ensure the correct functioning of the humbucker). The section was elevated to allow room for the humbucker, and so that the string could be angled downward at the bridge to simulate the correct conditions in the electric guitar. The supporting rods for the elevated sections were made of steel (O' Connor, 2011). Fig. 7.2 illustrates the support structure.

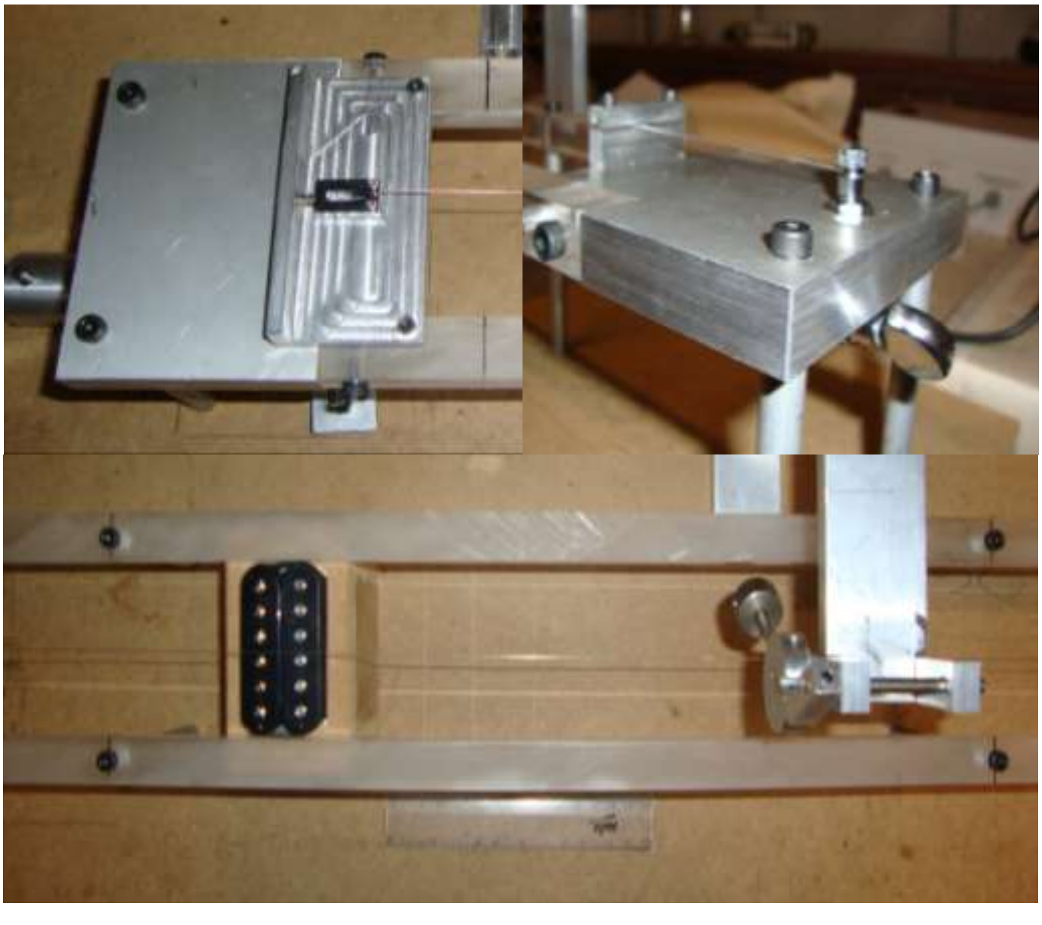

Figure 7.2 – Clockwise from top left: (i) bridge end plate, (ii) nut end plate and (iii) support beam sections.

7.1.3 Actuator

The actuator was an LT501150P model, manufactured by the Linear Master company (O' Neill, 2011). The load capacity for the device was 240N and it had a speed of 32mm/sec at zero load, drawing a supply voltage of 12 volts using direct current. It was mounted on the MDF base plate at the bridge end (see Fig. 7.3) and connected to the load cell which was attached to the string.

The movement of the actuator was input through LabVIEW 2010 software. Power was supplied to the actuator over a certain time interval, which could be adjusted prior to actuation. There existed a linear relationship between the degree of translation occurring and the time interval over which power was supplied, allowing for precise control over the actuator.

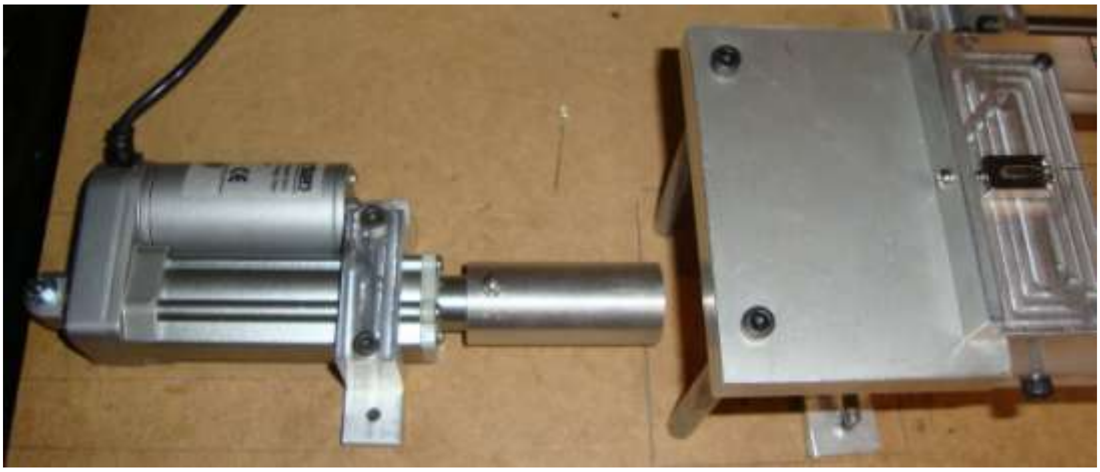

Figure 7.3 – Actuation device for the test rig.

7.1.4 Frequency Monitoring

An Intelli IMT 500 Digital Tuner (Fig. 7.4) was used to measure the fundamental frequency of the oscillating string. The tuner provided readings relative to standard musical notation which were then converted to provide the corresponding frequency results in hertz. This was attached to the rig at the aluminium bridge end plate using the clamp at the base of the tuner. The advantages of using this model were that it was wireless and could accurately read the frequency of the string oscillations irrespective of ambient noise in the surrounding environment.

Figure 7.4 – Intelli IMT 500 Digital Tuner (Intelli 2011).

7.1.6 Impulse Mechanism

A mechanical pendulum device was designed to impulse the string (Fig. 7.5). The device could be placed at any position along the scale length of the string to measure any variation in response. To impulse the string, the pendulum was first set to an initially static position, creating a fixed angle of rotation between the device and the string. The pendulum was then released and allowed to swing freely, striking the string and causing it to vibrate. To prevent the pendulum swinging backwards to reach equilibrium, striking the string a second time, a catch was installed above the fulcrum of the device to lock it in place after it had rotated through the necessary angle required to initiate the excitation (O' Connor, 2011).

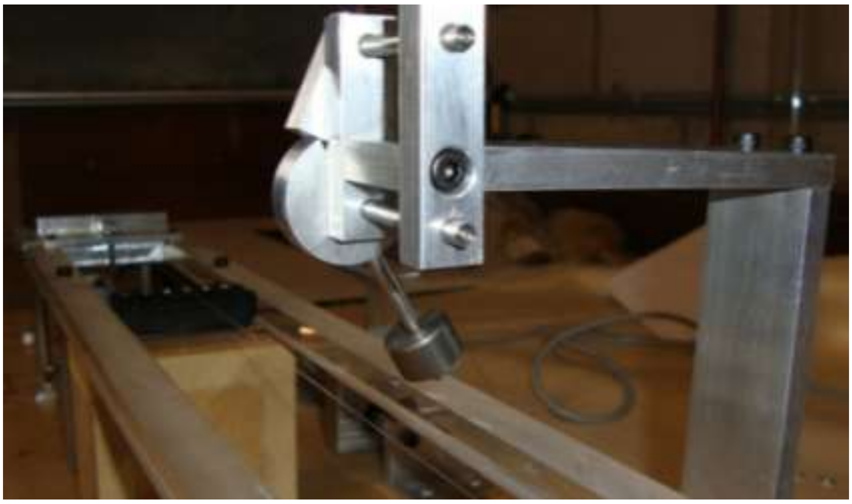

Figure 7.5 – Mechanical pendulum device used to impulse string.

7.1.7 Humbucker

The movement of the oscillating strings magnetic field induced a current due to electromagnetic induction in the humbucker (Fig. 7.6). This device consisted of two rows of bar magnets coiled in wire. It was placed in close proximity to the string underneath its scale length in order to register the frequency of the vibrations, which could be output as an electrical signal through an amplifier and loudspeaker to produce a sound of greater volume.

Figure 7.6 – Humbucker used to output frequency signal to amplifier.

7.2 Experimental Procedure

Two tests were performed using the experimental rig. The first test measured the creep rate of the string. This was carried out by measuring the extension occurring in the string as the tension was increased. The second test measured the tension acting on the string as the fundamental frequency was increased and decreased from standard tuning over a range of semitones. The purpose of the tests was to attempt to validate the rig design, as well as to validate the concept behind the group project. Details of this procedure can be found in Appendix B of this report.

8. Results and Discussion

8.1 Comparison of theoretical, finite element and experimental results

8.1.1 Tension versus Extension

The tension and extension present in the guitar string was measured experimentally from the test rig as the frequency was varied from 196Hz (G_3) to 311Hz ($D\#_4$) in increments of increasing semitones (O' Connor 2011, O'Neill 2011). The experimental extensions were recorded relative to an initial base value representing the distance between the adjustable end of the actuator and the fixed main body at the beginning of the test. These base values were 30.6mm and 30.05mm respectively for test 1 and test 2. The offset between these starting values was corrected in order to compare the resulting trends, so that both tests were plotted relative to the same base value of 30.05mm. The results were graphed against the values obtained from the finite element analysis (FEA) in Abaqus in addition to the results predicted from theoretical calculations, adjusting for this initial extension offset in the test rig to produce a measurable correlation (see Fig. 8.1).

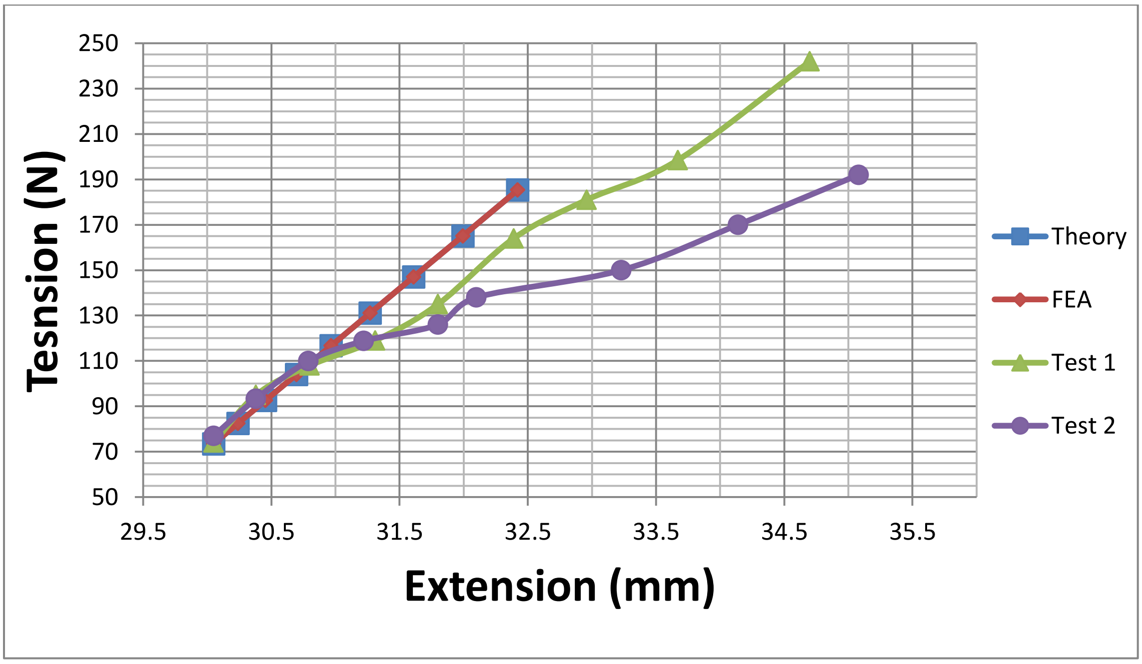

Figure 8.1 – Theoretical, FEA and experimental results for tension versus extension relationship.

The Abaqus model exhibited a linear relationship between tension and extension, producing a constant slope. These results correlated exactly with the theoretical calculations, with no margin of error. This relationship was expected to continue as long as the string was not subjected to stresses inducing strain in excess of the elastic limit of the material (the yield strength of the material was approximately 2000MPa).

The experimental results initially showed good correlation with this data, before deviating for higher frequencies. Greater than expected tension readings were measured from the load cell for increasing semitones, resulting in higher extension readings as the string was subjected to increased strain. The fact that the string was vibrating at the same frequencies despite sustaining higher than expected magnitudes of tension suggested that the scale length of the string was not being subjected to the same tension as was being measured by the load cell. Two factors were suggested as possible explanations for this discrepancy. The first factor was that the base plate of the rig was fabricated from medium-density fibre board. As this was a lightweight and flexible material, it was likely that the stresses existing in the rig induced a bending moment in the base plate which resulted in a source of error for the results. The second factor was that the string passed two contact points before being connected to the load cell, at the bridge and at a small steel bar that served to bend the string so that it could be connected to the load cell horizontally (this ensured that the tension in the string did not have a significant component existing in the y-direction, as the actuator was not designed to support vertical loads). It was suggested that increasing levels of friction developing between the steel bar and the string at higher tensions may have resulted in the string being subjected to higher stresses between the bar and the load cell as the actuator was displaced, and lower stresses between the bar and the nut. This would have had the effect of producing lower than expected frequencies for higher tension readings.

8.1.2 Tension versus Fret Board Semitones

The experimental relationship between the tension in the string and its corresponding frequency was investigated as the frequency was increased in semitones from $C\#_3$ to E_4. In order to produce an accurate reflection of the results, the test was carried out multiple times for each semitone to eliminate the impact of bad individual data points. The resulting relationship was plotted against the results predicted from the theoretical calculations and the finite element simulation (which were determined over the range of E_2 to G_4), shown in Fig. 8.2.

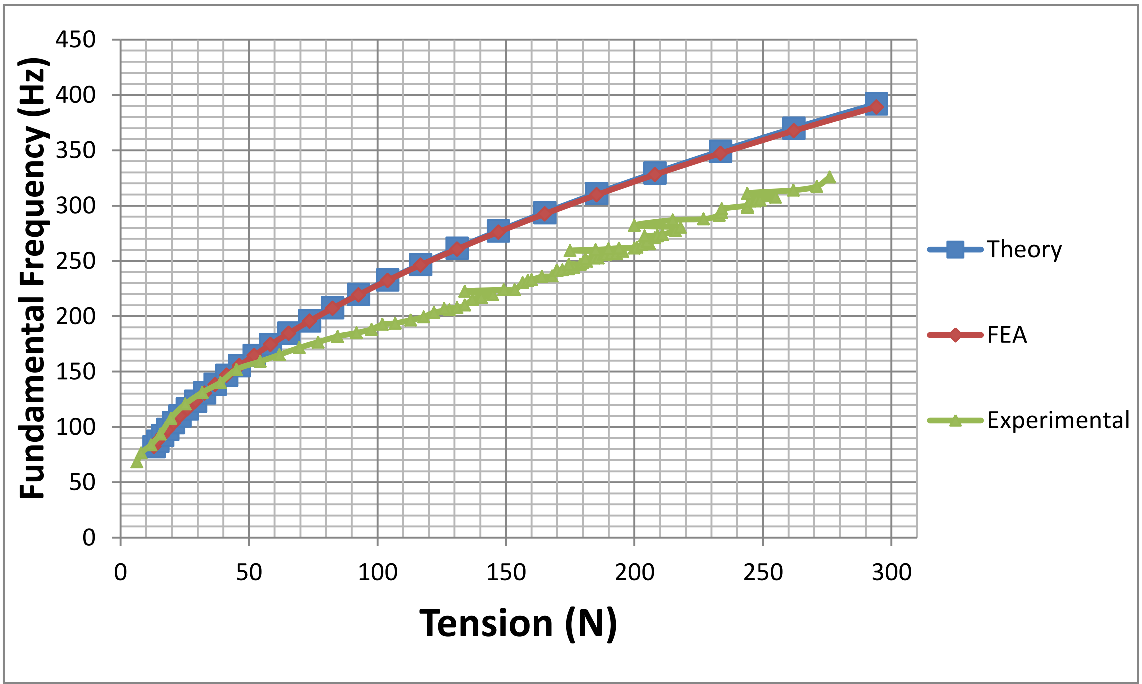

Figure 8.2 – Tension in string as frequency was increased in semitones.

The frequency increased as the tension was increased in the Abaqus model, producing a curved relationship. A sharp initial rate of increase in fundamental frequency as a result of relatively small increases in tension gradually began to level off as the tension reached greater magnitudes. At the peak of the graph, the slope of the curve had reduced considerably, as the frequency response of the string became more diminished. These results correlated closely with the theoretical calculations, with a small but gradually increasing margin of error over the range of the results. This increase in margin of error was attributed to the gradual extension of the model as the tension was increased. The resulting stress induced elastic strain along the longitudinal axis of the string, increasing its scale length. This change in scale length caused the fundamental frequency of the string to appear slightly lower than expected, as predicted from the relationship derived in Eq. 4.1.

The experimental results correlated closely with the predicted results until the tension was increased past approximately 45N. The results began to deviate for higher tensions due to the same reasons outlined in section 8.1.1. The friction between the string and the steel bar became less negligible as the tension was increased to higher magnitudes, resulting in a widening discrepancy for increasing values.

8.2 Comparison between theoretical and finite element results

8.2.1 Fundamental Frequency and Natural Harmonics for Standard Tuning

The fundamental frequency of the Abaqus model was determined when conditions present in the string at its standard tuning of G were simulated. The first 9 natural overtones of this fundamental were also produced and compared with the expected theoretical results, shown in Fig 8.3.

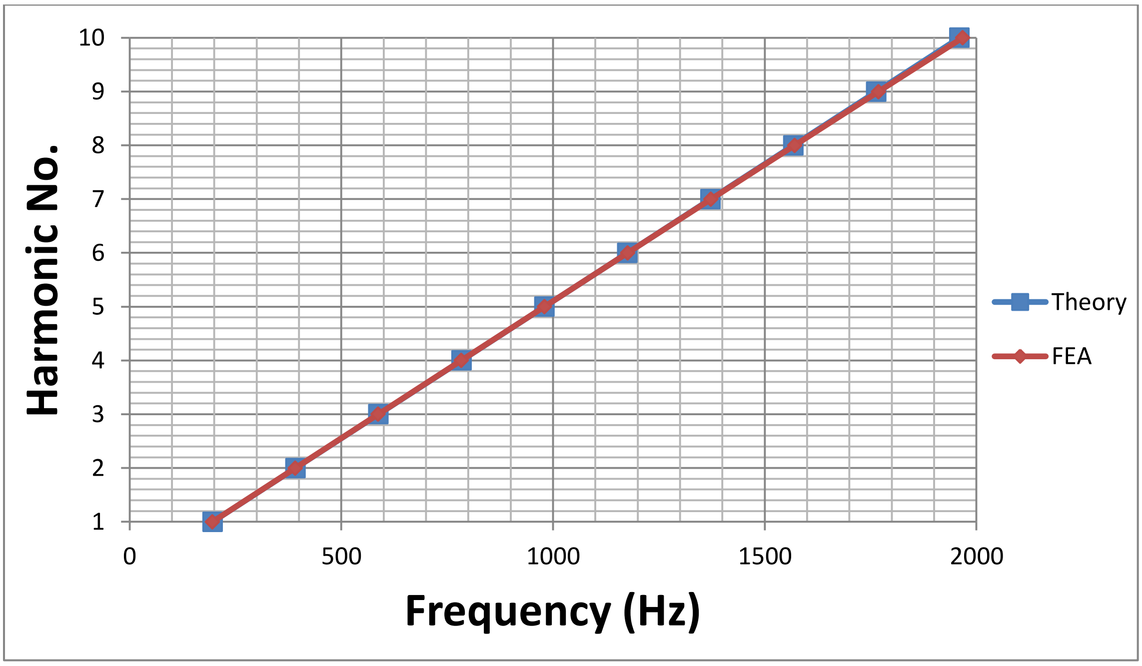

Figure 8.3 – Correlation between theoretical and FEA modal analysis results at standard tuning.

The fundamental frequency of the simulated model in standard tuning was determined to be 195.67Hz, which correlated very closely to the theoretical value of 196Hz. The subsequent harmonics of this fundamental abided by the results predicted from Eq. 3.14. The margin of error between the finite element analysis values and the corresponding theoretical values varied between 0.17% and -0.38%. The extension of the string due to the tensile stress it was subjected to resulted in an increase in scale length, effecting a slight reduction in the accuracy of the models frequency response.

The fundamental frequency and its harmonics had two corresponding modes of vibration each, as the string was free to oscillate in the xy and xz planes. Possible effects attributed to the rotation of the string about its longitudinal axis (x-axis) had no impact on the modal analysis results obtained from the model as this degree of freedom was constrained in the analysis.

8.2.2 Tension versus Fundamental Frequency

The fundamental frequency response of the string was evaluated in the Abaqus model as the tension was increased in fixed increments of 10N over a range of 0N to 200N. The modal analysis was run for each increment of tension, producing the fundamental mode of vibration of the string and its corresponding frequency. The simulated response was plotted against the results predicted from theoretical calculations, shown in Fig. 8.4. Fixing the tension increments illustrated the effect of tension on the strings frequency more accurately than the disproportional plot produced in section 8.1.2 which was based on increasing semitones (semitones do not increase in fixed frequency increments).

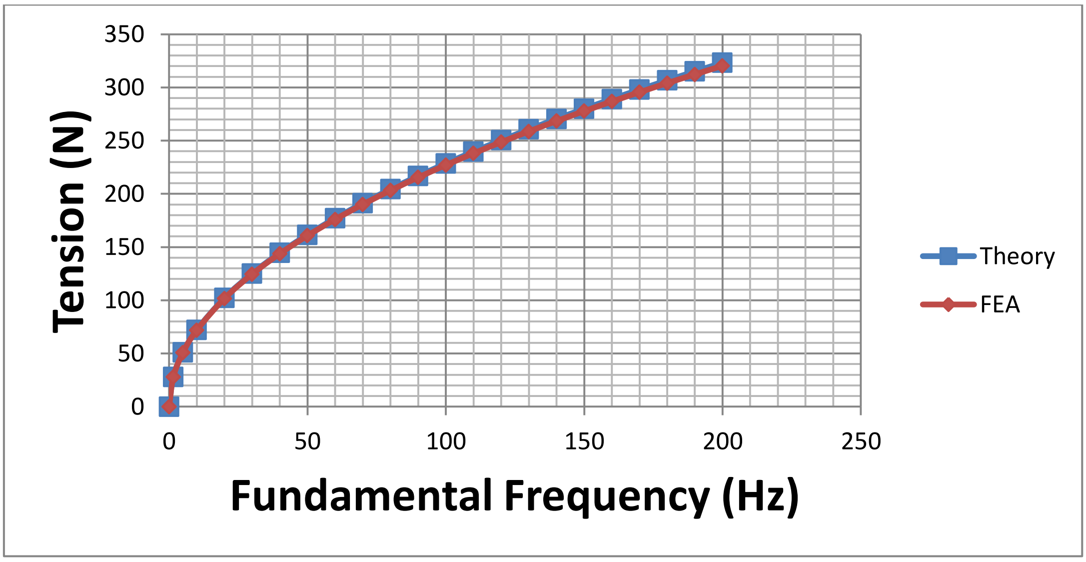

Figure 8.4 – Tension versus fundamental frequency for fixed increments of tension.

The fundamental frequency of the model increased as the tension was increased, producing a non-linear relationship. Initially small increases in tension elicited a sharp frequency response, showing an increase of 71.979Hz for the first 10N load. Additional data points were added at $T = 1.5$N and $T = 5$N to more clearly illustrate this phase. The slope of the graph saw a steep drop when the tension was doubled from 10N to 20N, giving rise to a frequency increase of 29.77Hz. As the curve began to level off, the slope of the graph gradually approached zero yielding an increasingly diminished frequency response for higher magnitudes of tension.

This relationship correlated closely with the results calculated from theory within a small margin of error. The error was 0.14% at 1.5N, increasing to 0.9% for 200N. Similarly to the relationship produced in section 8.1.2, this increase in margin of error was attributed to the gradual extension of the model as the tension was increased, resulting in a longer string scale length and a lower than expected fundamental frequency.

8.2.3 Linear Density versus Fundamental Frequency

The change in the fundamental frequency of the string was determined in Abaqus as the linear density was varied. The density of the simulated carbon steel material was increased in increments of 500 kg/m^3 from 0 kg/m^3 to 10000 kg/m^3. The linear increase of the material density corresponded to a linear increase in the linear density of the model in increments of 7.73 x 10^{-5} kg/m. These values were converted to grams per metre (g/m) for convenience and the frequency response obtained from the simulation was plotted against the corresponding results produced from theoretical calculations, displayed in Fig. 8.5.

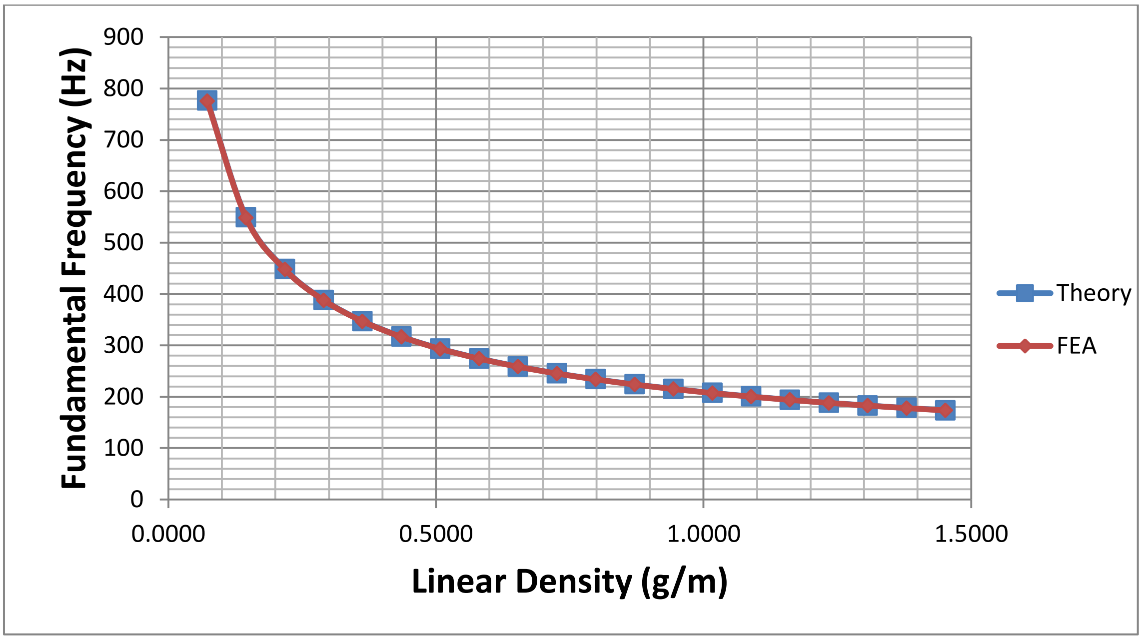

Figure 8.4 – Variation in fundamental frequency for increasing linear density.

The model exhibited an inverse relationship with increasing linear density producing a decrease in fundamental frequency. The plot was non-linear, showing a sharp initial drop in frequency of 227.09Hz for the first increment, before the slope gradually began to level off and approach zero. Higher magnitudes of linear density therefore saw a much smaller change in frequency, with a decrease of 4.51Hz for the final increment. This suggested that the string was very sensitive to change at low magnitudes of linear density, but was much less sensitive over the range of linear densities present in the materials typically used to manufacture electric guitar strings (between approximately 1.11 g/m and 1.16 g/m).

This relationship correlated very closely with the results predicted from theory within a negligible margin of error. This error varied within a range of 0.167% to 0.171% for each data point, remaining almost constant throughout the procedure.

8.2.4 Nodal deflection during vibration

The deflection of the central node of the model was recorded over the first 5 cycles of oscillation when an initial excitation produced an undamped free vibration in the string. The analysis was run undamped as the purpose of the test was not to measure the decay rate of the vibration (the time taken for a free vibration to damp out). As the string was vibrating at a rate of approximately 196 cycles per second, readings were taken every 2.55×10^{-4} seconds in order to produce approximately 20 position values for every cycle. This was run for 100 increments (5 cycles) and plotted in Fig. 8.5.

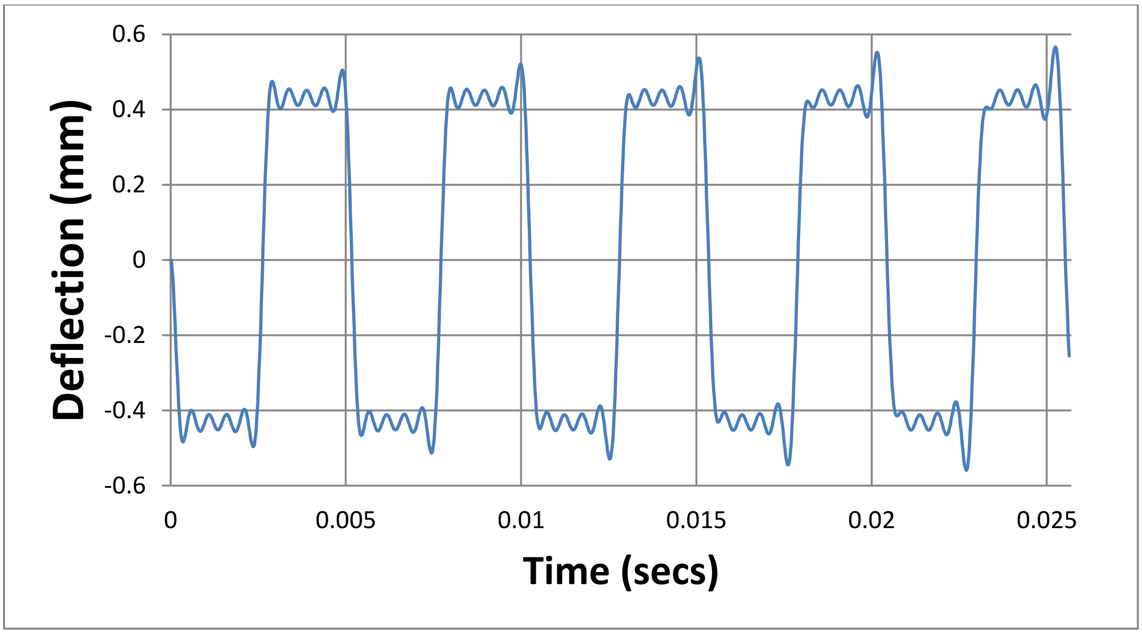

Figure 8.5 – Deflection of central node for first 5 oscillations following an initial excitation.

The load was applied at $t = 0$ seconds, and instantaneously removed after the first time increment in order to set the beginning of the free vibration at zero seconds. The applied force was approximated from the mass of the experimental pendulum (200 grams) to be 1.962N. This produced a deflection which reached an initial fundamental mode peak at 0.48mm in the negative y direction. As the string vibrated at its fundamental mode it simultaneously progressed through a pattern of its natural overtones, before approaching the opposite peak at 0.475mm in the positive y direction and repeating the same process. The analysis was run for 21 eigenvalues, ensuring that an accurate reflection of the most prominent modes of vibration was reflected in the harmonic pattern produced by the excitation.

9. Conclusions and Recommendations for Future Work

9.1 Conclusions

- Design parameters for the construction of an experimental test rig were defined by the results of a mathematical analysis of a vibrating guitar string
- A finite element simulation of the string was carried out, and a modal analysis test yielded the correct fundamental frequency of the string in standard tuning in addition to the sequence of its natural overtones
- The simulation was used to determine the relationships between the strings fundamental frequency and its governing parameters, producing results that correlated very closely with the values predicted from the mathematical analysis
- The deflection of the string was determined for a simulated plucking force, and the resulting harmonic pattern of vibration was analysed
- The role of project manager was fulfilled successfully, ensuring that the three aspects of the project ran in unison with each other and that all project tasks were completed on time

9.2 Recommendations for Future Work

- The finite element analysis model did not take into account the bending stresses that the string was subjected to at the bridge and steel bar junctions in the experimental test rig. A more extensive validation of the rig could take into account these stresses, in addition to the bending stresses existing in the base plate when high magnitudes of tension were applied to the string
- The deflection of the string can be recorded at various positions along the scale length and the results tested experimentally by filming the oscillating string with a high speed camera
- The decay rate of the free vibration can be measured by applying damping to the simulation. By comparing the simulation results to the decay rate of the experimental string it would be possible to determine what percentage of the energy losses during decay are attributable to the test rig design
- Future endeavours should seek to develop the concept tested in the group project and design a fully automated tremolo device for potential commercial applications

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Appendix A – Turnitin Originality Report

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Appendix B – Full Abaqus Procedure

- Abaqus version 6.9-1 was opened and the "create model database" option was selected to begin a new procedure.
- In the "Part" module, "Create Part" was selected to start modelling the guitar string. The string was defined as a 3D deformable wire of planar type. The approximate size value was set as 1. This was an arbitrary value which reflected the relative magnitude of the part's greatest dimension (in this case, the scale length of the string).
- In sketch mode, the string was drawn as a line using the "Create Lines: Connected" tool and the "Add Dimension" option allowed for the length of the string to be defined as 0.648m. (Abaqus uses arbitrary units so it is up to the user to remain consistent with all measurements. This entire analysis was performed using S.I. units.)
- Using the "Partition Edge: Parameter" option, the part was partitioned into 4 sections of equal length along the wire for use later on in the analysis.
- In the "Property" module, a new material was created using the "Create Material" option. The material was named "Commercially Pure Nickel" and given a density of $7850 kg/m^3$ in "Density" under the "General" tab. Under the "Mechanical" tab, the "Elasticity" option was expanded and "Elastic" was selected. The elastic modulus of the material was set as 210.29×10^9 Pa and the Poisson's ratio was defined as 0.313.
- Using the "Assign Beam Orientation" tool, and the highlighting the part, the string was given its orientation. The default values for the orientation were accepted.
- In "Create Section", a section in the category "Beam" of type "Beam" was chosen for the profile of the string. A circular profile of radius 0.215×10^{-3}m was then created under the name "String Profile". This defined the circular cross section of the string, as well as the magnitude of the string gauge (i.e. its diameter). This section was then applied to the part using the "Assign Section" tool and highlighting the part.
- In the "Assembly" module, using the "Instance Part" button, the part was assembled with an independent instance type. (As there was only one component in the model, no actual assembly process was required.)

- In the “Step” module, using the “Create Step” tool, the “Static, General” option was selected under the “General” procedure type to create a new step after the initial one.
- In the “Load” module, in “Create Boundary Condition”, the boundary conditions were set at both end nodes of the string for Step 1, representing the nut and bridge of the guitar. Selecting the node at the bridge end, displacements in the U1, U2, and U3 direction in addition to rotations about the U3 axis (UR3) were set to zero. This left the string free to rotate about the U1 and U2 axes, reducing the bridge node from 6 degrees of freedom to 2. At the nut node, displacements in the U2 and U3 direction in addition to rotations about the U3 axis (UR3) were set to zero, but a nodal displacement representing the extension of the string in the U1 axis due to its tension was set. This had a value of 1.73312×10^{-3}m for a standard tuned G string.
- The “Step” module was returned to, and a “Frequency” step was created under the “Linear Perturbation” procedure type in order to perform a modal analysis. 21 eigenvalues were requested for the analysis in order to calculate the fundamental frequency of the string as well as its first 9 natural overtones.
- In the “Step Manager” window, the “Nlgeom” button was selected and the boxes were checked beside steps 1 and 2. This ensured that the nodal displacement from step 1 would be carried on to step 2 in order to calculate the correct frequency of the string in tension during the modal analysis.
- In the “Load” module, a new pair of boundary conditions was created using the same process as before at the nut and bridge nodes. Displacements in the U1, U2, and U3 direction in addition to rotations about the U3 axis (UR3) were set to zero for both of these nodes.
- Returning once again to the “Step” module, a final step was created by selecting “Modal Dynamics” under the “Linear Perturbation” procedure type. Over a time period of 1 second, a time increment of 0.0002551 seconds was specified in order to produce a total of 3920 analysis frames in the resulting output.
- In the “Load” module, a concentrated force was created in the modal dynamics step named “Pluck” to represent the plucking impulse acting on the string. This was applied at the central node ($l = 0.324$m) of the string for the first analysis, followed by the nodes at $l = 0.162$m and $l = 0.486$m for subsequent analyses. In

order to validate the rig, the magnitude of the impulse was calculated to match that of the pendulum device used in the experimentation procedure. As the mass of the pendulum was approximately 0.2kg, the impulse was calculated to reach a maximum of about 1.962N when it struck the G string. This impulse was applied in the y-direction (application in the z-direction would have yielded the same result). A tabular amplitude profile was created to represent the impulse. At $t = 0$ seconds, the amplitude of the impulse was input as 1 to specify that the impulse was at its maximum amplitude at this time (i.e. maximum deflection reached at $t = 0$ seconds, initiating the first cycle from this position). The amplitude of the impulse was input to have reached 0 by the end of this initial time increment, at $t = 0.0002551$ seconds, and to remain at zero amplitude for the rest of the analysis step, allowing the string to vibrate freely after the initial excitation.

- In the "Mesh" module, the "Seed Edge: By Number" option was used to seed the model with a total of 800 seeds, divided equally across the partitions.
- "Assign Element Type" allowed the element type to be configured for the mesh. Standard linear beam elements were applied to the string model, linear elements being the optimum choice for wave problems as quadratic elements would distribute the mass of the string unevenly across the nodes for analyses.
- After the element type was confirmed, "Mesh Part Instance" was used to apply the chosen mesh to the part instance.
- In the "Job" module, a new job was created using the "Create Job" option. All default settings were accepted. The new job was right-clicked on in the model tree and "Submit" was chosen.
- When the submitted job was completed, the job was right clicked on again in the model tree, and "Results" was selected in order to display the results in the "Visualization" module. By selecting "Allow Multiple Plot States", followed by "Plot Deformed Shape" and "Plot Undeformed Shape", it was possible to view the shapes of the modes of vibration of the string for the modal analysis step as superimposed over the static, undeformed string in its initial state. It was possible to display the different modes of oscillation and their frequencies using the arrow buttons in the context bar above the canvas. By selecting "Plot Contours on Deformed Shape", the magnitudes of the string deformations at the

different positions along its length were displayed. The modal dynamics step produced 20 frames per cycle for the string in standard tuning, allowing the user to play an animation of the plucked string vibrating at a rate of 196Hz using the “Animate: Time History” option. The speed of the animation could be adjusted through the “Animation Options” window to provide the most ideal visualisation.

Appendix C – Test Procedures

Experimental Procedure 1 – Measuring Tension and Extension

1. The string was attached to the load cell, which was then screwed onto the actuator.
2. The actuator was positioned at the centre point of its range of motion to allow for maximum displacement in the positive and negative direction of the x axis, and the distance between the end of the actuator and its base was measured using the electronic vernier callipers. This value was recorded to provide a reference frame for the displacement of the actuator during the procedure.
3. The tensometer was connected to the load cell, and the initial tension in the string was recorded.
4. The string was set to its standard tuning of G_3 using the locking tuner, which had a corresponding fundamental frequency of 196Hz. This frequency was verified using the electronic tuner, and the string was clamped in place at the nut end.
5. In LabVIEW, the time interval over which the pulse would be applied to the actuator was set to a value of 10 milliseconds. This standardised the displacement of the actuator for the test.
6. A pulse was sent to the actuator to displace it in the positive x direction, increasing the tension in the string. The distance between the end of the actuator and its base was measured again using the callipers. The difference between this value and the initial value was recorded to provide the extension of the string for the first 10 millisecond pulse. The tension in the string was read from the tensometer, and the difference between this value and the initial value was recorded to provide the increase in tension.
7. Step 6 was repeated the desired number of times for the test, recording the increase in tension and extension after each pulse.
8. The actuator was returned to its initial position, and the string was tuned back down to standard tuning.
9. A pulse was sent to the actuator to displace it in the negative x direction, decreasing the tension in the string. As in step 6, the change in tension and extension was recorded after each pulse.
10. Step 9 was repeated the desired number of times for the test.
11. The change in tension was plotted against the change in extension.

Experimental Procedure 2 – Measuring Tension for Increasing Semitones

1. The string was attached to the load cell, which was then screwed onto the actuator.
2. The actuator was positioned at the centre point of its range of motion to allow for maximum displacement in the positive and negative direction of the x axis.
3. The string was set to its standard tuning of G_3 using the locking tuner, which had a corresponding fundamental frequency of 196Hz. This frequency was verified using the electronic tuner, and the string was clamped in place at the nut end.
4. The tensometer was connected to the load cell, and the initial tension in the string was recorded.
5. In LabVIEW, pulses were sent to the actuator to displace it in the positive x direction, until the tuner read that the frequency of the string had been increased to the next semitone. The tension in the string was read from the tensometer, and the difference between this value and the initial value was recorded to provide the increase in tension.
6. Step 5 was repeated until the desired upper semitone of the test range was reached, recording the increase in tension after each pulse.
7. The actuator was returned to its initial position, and the string was tuned back down to standard tuning.
8. Pulses were sent to the actuator to displace it in the negative x direction, decreasing the frequency of the string through a range of lower semitones. As in step 5, the change in tension was recorded at each semitone.
9. Step 8 was repeated until the lower semitone of the test range was reached.
10. The change in tension was plotted against the change in extension.