The leakage inductances L_1 and L_2 are usually not crucial to the general operation of the power electronics circuits described in this chapter, but they are important when considering switching transients. Note that in ac power system applications, the leakage inductance is normally the important analysis and design parameter.

For periodic voltage and current operation for a transformer circuit, the magnetic flux in the core must return to its starting value at the end of each switching period. Otherwise, flux will increase in the core and eventually cause saturation. A saturated core cannot support a voltage across a transformer winding, and this will lead to device currents that are beyond the design limits of the circuit.

7.3 THE FLYBACK CONVERTER

Continuous-Current Mode

A dc-dc converter that provides isolation between input and output is the flyback circuit of Fig. 7-2*a*. In a first analysis, Fig. 7-2*b* uses the transformer model which includes the magnetizing inductance L_m, as in Fig. 7-1*d*. The effects of

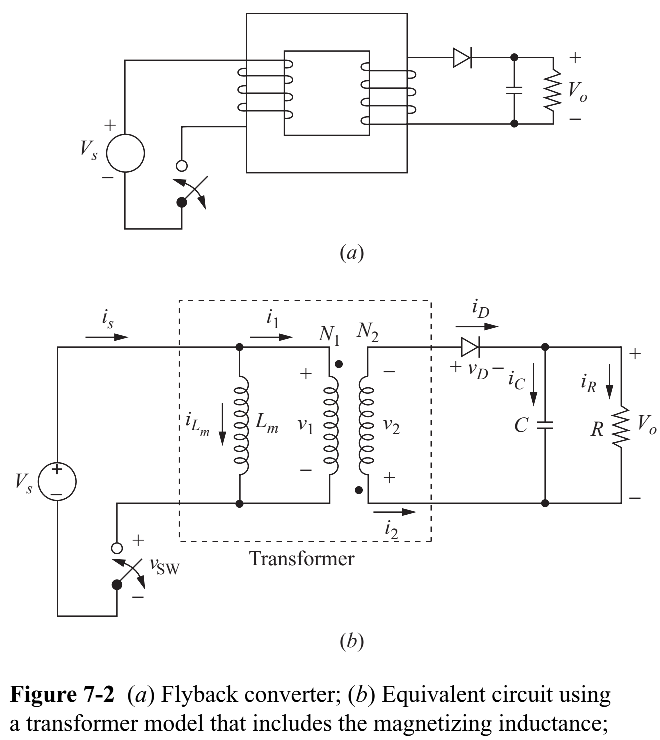


Figure 7-2 (*a*) Flyback converter; (*b*) Equivalent circuit using a transformer model that includes the magnetizing inductance; (*c*) Circuit for the switch on; (*d*) Circuit for the switch off.

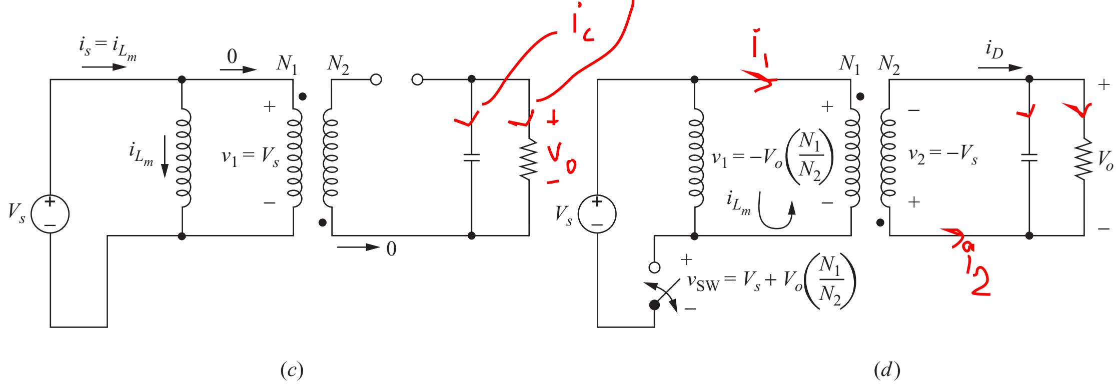


Figure 7-2 (*continued*)

losses and leakage inductances are important when considering switch performance and protection, but the overall operation of the circuit is best understood with this simplified transformer model. Note the polarity of the transformer windings in Fig. 7-2.

Additional assumptions for the analysis are made:

1. The output capacitor is very large, resulting in a constant output voltage V_o.
2. The circuit is operating in the steady state, implying that all voltages and currents are periodic, beginning and ending at the same points over one switching period.
3. The duty ratio of the switch is D, being closed for time DT and open for $(1 - D)T$.
4. The switch and diode are ideal.

The basic operation of the flyback converter is similar to that of the buck-boost converter described in Chap. 6. Energy is stored in L_m when the switch is closed and is then transferred to the load when the switch is open. The circuit is analyzed for both switch positions to determine the relationship between input and output.

Analysis for the Switch Closed On the source side of the transformer (Fig. 7-2*c*),

$$v_1 = V_s = L_m \frac{di_{L_m}}{dt}$$

$$\frac{di_{L_m}}{dt} = \frac{\Delta i_{L_m}}{\Delta t} = \frac{\Delta i_{L_m}}{DT} = \frac{V_s}{L_m}$$

Solving for the change in current in the transformer magnetizing inductance,

$$(\Delta i_{L_m})_{\text{closed}} = \frac{V_s DT}{L_m} \tag{7-2}$$

On the load side of the transformer,

$$v_2 = v_1\left(\frac{N_2}{N_1}\right) = V_s\left(\frac{N_2}{N_1}\right)$$

$$v_D = -V_o - V_s\left(\frac{N_2}{N_1}\right) < 0$$

$$i_2 = 0$$

$$i_1 = 0$$

Since the diode is off, $i_2 = 0$, which means that $i_1 = 0$. So while the switch is closed, current is increasing linearly in the magnetizing inductance L_m, and there is no current in the windings of the ideal transformer in the model. Remember that in the actual transformer, this means that the current is increasing linearly in the physical primary winding, and no current exists in the secondary winding.

Analysis for the Switch Open When the switch opens (Fig. 7-2*d*), the current cannot change instantaneously in the inductance L_m, so the conduction path must be through the primary turns of the ideal transformer. The current i_{L_m} enters the undotted terminal of the primary and must exit the undotted terminal of the secondary. This is allowable since the diode current is positive. Assuming that the output voltage remains constant at V_o, the transformer secondary voltage v_2 becomes $-V_o$. The secondary voltage transforms back to the primary, establishing the voltage across L_m at

$$v_1 = -V_o\left(\frac{N_1}{N_2}\right)$$

Voltages and currents for an open switch are

$$v_2 = -V_o$$

$$v_1 = v_2\left(\frac{N_1}{N_2}\right) = -V_o\left(\frac{N_1}{N_2}\right)$$

$$L_m\frac{di_{L_m}}{dt} = v_1 = -V_o\left(\frac{N_1}{N_2}\right)$$

$$\frac{di_{L_m}}{dt} = \frac{\Delta i_{L_m}}{\Delta t} = \frac{\Delta i_{L_m}}{(1-D)T} = \frac{-V_o}{L_m}\left(\frac{N_1}{N_2}\right)$$

Solving for the change in transformer magnetizing inductance with the switch open,

$$(\Delta i_{L_m})_{\text{open}} = \frac{-V_o(1-D)T}{L_m}\left(\frac{N_1}{N_2}\right) \tag{7-3}$$

Since the net change in inductor current must be zero over one period for steady-state operation, Eqs. (7-2) and (7-3) show

$$(\Delta i_{L_m})_{\text{closed}} + (\Delta i_{L_m})_{\text{open}} = 0$$

$$\frac{V_s DT}{L_m} - \frac{V_o(1-D)T}{L_m}\left(\frac{N_1}{N_2}\right) = 0$$

Solving for V_o,

$$\boxed{V_o = V_s\left(\frac{D}{1-D}\right)\left(\frac{N_2}{N_1}\right)} \tag{7-4}$$

Note that the relation between input and output for the flyback converter is similar to that of the buck-boost converter but includes the additional term for the transformer ratio.

Other currents and voltages of interest while the switch is open are

$$\begin{aligned} i_D &= -i_1\left(\frac{N_1}{N_2}\right) = i_{L_m}\left(\frac{N_1}{N_2}\right) \\ v_{\text{sw}} &= V_s - v_1 = V_s + V_o\left(\frac{N_1}{N_2}\right) \\ i_R &= \frac{V_o}{R} \\ i_C &= i_D - i_R = i_{L_m}\left(\frac{N_1}{N_2}\right) - \frac{V_o}{R} \end{aligned} \tag{7-5}$$

Note that v_{sw}, the voltage across the open switch, is greater than the source voltage. If the output voltage is the same as the input and the turns ratio is 1, for example, the voltage across the switch will be twice the source voltage. Circuit currents are shown in Fig. 7-3.

The power absorbed by the load resistor must be the same as that supplied by the source for the ideal case, resulting in

$$P_s = P_o$$

or

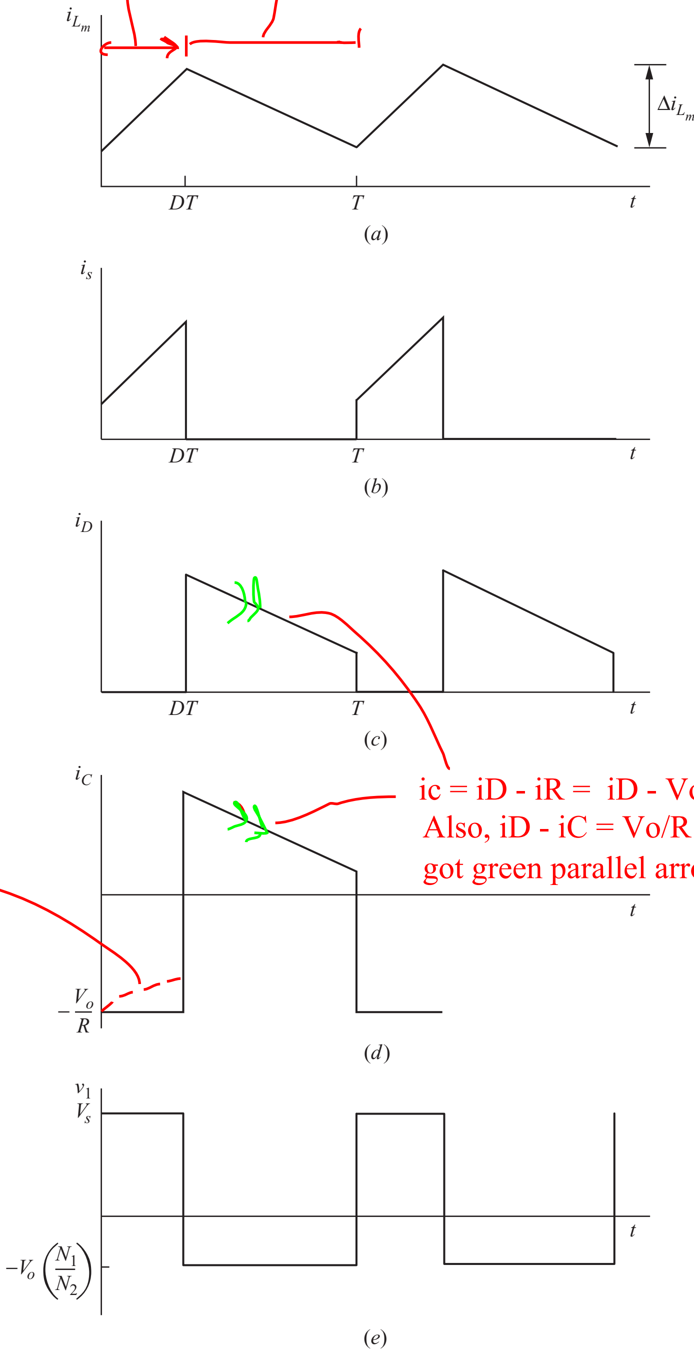
$$V_s I_s = \frac{V_o^2}{R} \tag{7-6}$$

The average source current I_s is related to the average of the magnetizing inductance current I_{L_m} by

$$I_s = \frac{(I_{L_m})DT}{T} = I_{L_m}D \tag{7-7}$$

is = iLm

i1= -iLm
= -(N2/N1)*iD



Actually, there should be discharging like this

ic = iD - iR = iD - Vo/R = -(N1/N2)i1 - Vo/R
Also, iD - iC = Vo/R at any time, thereby we got green parallel arrows.

Figure 7-3 Flyback converter current and voltage waveforms. (*a*) Magnetizing inductance current; (*b*) Source current; (*c*) Diode current; (*d*) Capacitor current; (*e*) Transformer primary voltage.

Substituting for I_s in Eq. (7-6) and solving for I_{L_m},

$$V_s I_{L_m} D = \frac{V_o^2}{R}$$
$$I_{L_m} = \frac{V_o^2}{V_s D R} \tag{7-8}$$

Using Eq. (7-4) for V_s, the average inductor current is also expressed as

$$I_{L_m} = \frac{V_s D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 = \frac{V_o}{(1-D)R}\left(\frac{N_2}{N_1}\right) \tag{7-9}$$

The maximum and minimum values of inductor current are obtained from Eqs. (7-9) and (7-2).

$$\begin{aligned} I_{L_{m,\max}} &= I_{L_m} + \frac{\Delta i_{L_m}}{2} \\ &= \frac{V_s D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 + \frac{V_s D T}{2L_m} \end{aligned} \tag{7-10}$$

$$\begin{aligned} I_{L_{m,\min}} &= I_{L_m} - \frac{\Delta i_{L_m}}{2} \\ &= \frac{V_s D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 - \frac{V_s D T}{2L_m} \end{aligned} \tag{7-11}$$

Continuous-current operation requires that $I_{L_{m,\min}} > 0$ in Eq. (7-11). At the boundary between continuous and discontinuous current,

$$I_{L_{m,\min}} = 0$$
$$\frac{V_s D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 = \frac{V_s D T}{2L_m} = \frac{V_s D}{2L_m f}$$

where f is the switching frequency. Solving for the minimum value of L_m that will allow continuous current,

$$\boxed{(L_m)_{\min} = \frac{(1-D)^2 R}{2f}\left(\frac{N_1}{N_2}\right)^2} \tag{7-12}$$

In a flyback converter design, L_m is selected to be larger than $L_{m,\min}$ to ensure continuous current operation. A convenient expression relating inductance and current variation is found from Eq. (7-2).

$$L_m = \frac{V_s D T}{\Delta i_{L_m}} = \frac{V_s D}{\Delta i_{L_m} f} \tag{7-13}$$

The output configuration for the flyback converter is the same as for the buck-boost converter, so the output ripple voltages for the two converters are also the same.

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{D}{RCf}} \tag{7-14}$$

As with the converters described in Chap. 6, the equivalent series resistance of the capacitor can contribute significantly to the output voltage ripple. The peak-to-peak variation in capacitor current is the same as the maximum current in the diode and the transformer secondary. Using Eq. (7-5), the voltage ripple due to the ESR is

$$\Delta V_{o,\mathrm{ESR}} = \Delta i_C r_C = I_{L_{m,\max}}\left(\frac{N_1}{N_2}\right) r_C \tag{7-15}$$

EXAMPLE 7-1

Flyback Converter

A flyback converter of Fig. 7-2 has the following circuit parameters:

$V_s = 24$ V
$N_1/N_2 = 3.0$
$L_m = 500\ \mu$H
$R = 5\ \Omega$
$C = 200\ \mu$F
$f = 40$ kHz
$V_o = 5$ V

Determine (*a*) the required duty ratio D; (*b*) the average, maximum, and minimum values for the current in L_m; and (*c*) the output voltage ripple. Assume that all components are ideal.

■ Solution

(*a*) Rearranging Eq. (7-4) yields

$$V_o = V_s\left(\frac{D}{1-D}\right)\left(\frac{N_2}{N_1}\right)$$

$$D = \frac{1}{(V_s/V_o)(N_2/N_1) + 1} = \frac{1}{(24/5)(1/3) + 1} = 0.385$$

(*b*) Average current in L_m is determined from Eq. (7-8).

$$I_{L_m} = \frac{V_o^2}{V_s DR} = \frac{5^2}{(24)(0.385)(5)} = 540 \text{ mA}$$

The change in i_{L_m} can be calculated from Eq. (7-2).

$$\Delta i_{L_m} = \frac{V_s D}{L_m f} = \frac{(24)(0.385)}{500(10)^{-6}(40{,}000)} = 460 \text{ mA}$$

Maximum and minimum inductor currents can be computed from

$$I_{L_{m,\max}} = I_{L_m} + \frac{\Delta i_{L_m}}{2} = 540 + \frac{460}{2} = 770 \text{ mA}$$

$$I_{L_{m,\min}} = I_{L_m} - \frac{\Delta i_{L_m}}{2} = 540 - \frac{460}{2} = 310 \text{ mA}$$

Equations (7-10) and (7-11), which are derived from the above computation, could also be used directly to obtain the maximum and minimum currents. Note that a positive $I_{Lm,\min}$ verifies continuous current in L_m.

(*c*) Output voltage ripple is computed from Eq. (7-14).

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} = \frac{0.385}{(5)[200(10)^{-6}](40{,}000)} = 0.0096 = 0.96\%$$

EXAMPLE 7-2

Flyback Converter Design, Continuous-Current Mode

Design a converter to produce an output voltage of 36 V from a 3.3-V source. The output current is 0.1 A. Design for an output ripple voltage of 2 percent. Include ESR when choosing a capacitor. Assume for this problem that the ESR is related to the capacitor value by $r_C = 10^{-5}/C$.

■ Solution

Considering a boost converter for this application and calculating the required duty ratio from Eq. (6-27),

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{3.3}{36} = 0.908$$

The result of a high duty ratio will likely be that the converter will not function as desired because of losses in the circuit (Fig. 6-10). Therefore, a boost converter would not be a good choice. A flyback converter is much better suited for this application.

As a somewhat arbitrary design decision, start by letting the duty ratio be 0.4. From Eq. (7-4), the transformer turns ratio is calculated to be

$$\left(\frac{N_2}{N_1}\right) = \frac{V_o}{V_s}\left(\frac{1-D}{D}\right) = \frac{36}{3.3}\left(\frac{1-0.4}{0.4}\right) = 16.36$$

Rounding, let $N_2/N_1 = 16$. Recalculating the duty ratio using a turns ratio of 16 gives $D = 0.405$.

To determine L_m, first compute the average current in L_m from Eq. (7-9), using $I_o = V_o/R$.

$$I_{L_m} = \frac{V_o}{(1-D)R}\left(\frac{N_2}{N_1}\right) = \frac{I_o}{1-D}\left(\frac{N_2}{N_1}\right) = \left(\frac{0.1}{1-0.405}\right)16 = 2.69 \text{ A}$$

Let the current variation in L_m be 40 percent of the average current: $\Delta i_{L_m} = 0.4(2.69) = 1.08$ A. As another somewhat arbitrary choice, let the switching frequency be 100 kHz. Using Eq. (7-13),

$$L_m = \frac{V_s D}{\Delta i_{L_m} f} = \frac{3.3(0.405)}{1.08(100{,}000)} = 12.4\ \mu\text{H}$$

Maximum and minimum currents in L_m are found from Eqs. (7-10) and (7-11) as 3.23 and 2.15 A, respectively.

The output voltage ripple is to be limited to 2 percent, which is $0.02(36) = 0.72$ V. Assume that the primary cause of the voltage ripple will be the voltage drop across the equivalent series resistance $\Delta i_C r_C$. The peak-to-peak variation in capacitor current is the same as in the diode and the transformer secondary and is related to current in L_m by

$$\Delta i_C = I_{L_{m,\max}}\left(\frac{N_1}{N_2}\right) = (3.23\text{ A})\left(\frac{1}{16}\right) = 0.202\text{ A}$$

Using Eq. (7-15),

$$r_C = \frac{\Delta V_{o,\text{ESR}}}{\Delta i_C} = \frac{0.72\text{ V}}{0.202\text{ A}} = 3.56\ \Omega$$

Using the relationship between ESR and capacitance given in this problem,

$$C = \frac{10^{-5}}{r_C} = \frac{10^{-5}}{3.56} = 2.8\ \mu\text{F}$$

The ripple voltage due to the capacitance only is obtained from Eq. (7-14) as

$$\frac{\Delta V_o}{V_o} = \frac{D}{RCf} = \frac{0.405}{(36\text{ V}/0.1\text{ A})[2.8(10)^{-6}](100{,}000)} = 0.004 = 0.04\%$$

showing that the assumption that the voltage ripple is primarily due to the ESR was correct. A standard value of 3.3 μF would be a good choice. Note that the designer should consult manufacturers' specifications for ESR when selecting a capacitor.

The turns ratio of the transformer, current variation, and switching frequency were selected somewhat arbitrarily, and many other combinations are suitable.

Discontinuous-Current Mode in the Flyback Converter

For the discontinuous-current mode for the flyback converter, the current in the transformer increases linearly when the switch is closed, just as it did for the continuous-current mode. However, when the switch is open, the current in the transformer magnetizing inductance decreases to zero before the start of the next switching cycle, as shown in Fig. 7-4. While the switch is closed, the increase in inductor current is described by Eq. (7-2). Since the current starts at zero, the maximum value is also determined from Eq. (7-2).

$$I_{L_{m,\max}} = \frac{V_s DT}{L_m} \tag{7-16}$$

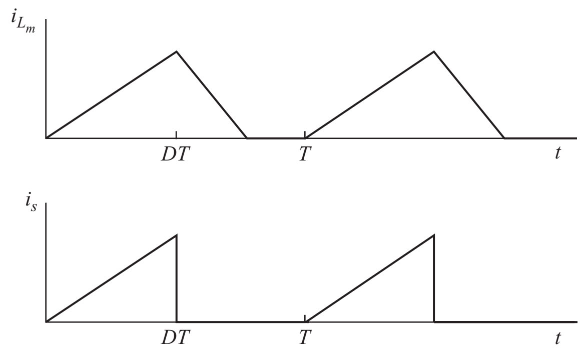


Figure 7-4 Discontinuous current for the flyback converter.

The output voltage for discontinuous-current operation can be determined by analyzing the power relationships in the circuit. If the components are ideal, the power supplied by the dc source is the same as the power absorbed by the load resistor. Power supplied by the source is the dc voltage times average source current, and load power is V_o^2/R:

$$\begin{aligned} P_s &= P_o \\ V_s I_s &= \frac{V_o^2}{R} \end{aligned} \tag{7-17}$$

Average source current is the area under the triangular waveform of Fig. 7-4*b* divided by the period, resulting in

$$I_s = \left(\frac{1}{2}\right)\left(\frac{V_s DT}{L_m}\right)(DT)\left(\frac{1}{T}\right) = \frac{V_s D^2 T}{2L_m} \tag{7-18}$$

Equating source power and load power [Eq. (7-17)],

$$\frac{V_s^2 D^2 T}{2L_m} = \frac{V_o^2}{R} \tag{7-19}$$

Solving for V_o for discontinuous-current operation in the flyback converter,

$$\boxed{V_o = V_s D \sqrt{\frac{TR}{2L_m}} = V_s D \sqrt{\frac{R}{2L_m f}}} \tag{7-20}$$

EXAMPLE 7-3

Flyback Converter, Discontinuous Current

For the flyback converter in Example 7-1, the load resistance is increased from 5 to 20 Ω with all other parameters remaining unchanged. Show that the magnetizing inductance current is discontinuous, and determine the output voltage.

■ **Solution**

Using $L_m = 500\ \mu\text{H}$, $f = 40$ kHz, $N_1/N_2 = 3$, $D = 0.385$, and $R = 20\ \Omega$, the minimum inductor current from Eq. (7-11) is calculated as

$$I_{L_{m,\min}} = \frac{V_s D}{(1-D)^2 R}\left(\frac{N_2}{N_1}\right)^2 - \frac{V_s DT}{2L_m}$$

$$= \frac{(24)(0.385)}{(1-0.385)^2(20)}\left(\frac{1}{3}\right)^2 - \frac{(24)(0.385)}{2(500)(10)^{-6}(40{,}000)} = -95 \text{ mA}$$

Since negative current in L_m is not possible, i_{L_m} must be discontinuous. Equivalently, the minimum inductance for continuous current can be calculated from Eq. (7-12).

$$(L_m)_{\min} = \frac{(1-D)^2 R}{2f}\left(\frac{N_1}{N_2}\right)^2 = \frac{(1-0.385)^2 20}{2(40{,}000)}(3)^2 = 850\ \mu\text{H}$$

which is more than the 500 μH specified, also indicating discontinuous current.
Using Eq. (7-20),

$$V_o = V_s D\sqrt{\frac{R}{2L_m f}} = (24)(0.385)\sqrt{\frac{20}{2(500)(10)^{-6}(40{,}000)}} = 6.53 \text{ V}$$

For the current in L_m in the discontinuous-current mode, the output voltage is no longer 5 V but increases to 6.53 V. Note that for any load that causes the current to be continuous, the output would remain at 5 V.

Summary of Flyback Converter Operation

When the switch is closed in the flyback converter of Fig. 7-2*a*, the source voltage is across the transformer magnetizing inductance L_m and causes i_{L_m} to increase linearly. Also while the switch is closed, the diode on the output is reverse-biased, and load current is supplied by the output capacitor. When the switch is open, energy stored in the magnetizing inductance is transferred through the transformer to the output, forward-biasing the diode and supplying current to the load and to the output capacitor. The input-output voltage relationship in the continuous-current mode of operation is like that of the buck-boost dc-dc converter but includes a factor for the turns ratio.

7.4 THE FORWARD CONVERTER

The forward converter, shown in Fig. 7-5*a*, is another magnetically coupled dc-dc converter. The switching period is T, the switch is closed for time DT and open for $(1 - D)T$. Steady-state operation is assumed for the analysis of the circuit, and the current in inductance L_x is assumed to be continuous.

The transformer has three windings: windings 1 and 2 transfer energy from the source to the load when the switch is closed; winding 3 is used to provide a path for the magnetizing current when the switch is open and to reduce the magnetizing current to zero before the start of each switching period. The transformer