hello

i solved the task not so sure about my answers , i'll be presenting this task to the white board, would you please check it out and tell me what should i add or correct and the way i'm explaining the task is correct ,enough or not ?

$p_0 = \frac{1}{3}$; $p_1 = (1-c)\,\frac{2}{3}$; $p_2 = c\,\frac{2}{3}$; (c $\in [0,1]$)

$q = \sum_{i=0}^{n} p_i q^i$, in general case

q = dying probability for process

$q = \sum_{i=0}^{2} p_i q^i$==> in our case X => is a random variable for i offspring.

Ex < 1 => q=1.
Ex > 1 => q < 1 - 2 .

so we have two to compute the critical value c^*

a) first case q=1 , Ex <1 .

Ex = $\sum_{i=0}^{2} X_i q^i$ = $0 \times \frac{1}{3} + 1\,(1-c)\,\frac{2}{3} + 2\,(c\,\frac{2}{3})$

$\frac{2}{3}(1-c) + \frac{4c}{3} < 1$

$2 - 2c + 4c < 3$

C < $\frac{1}{2}$, c<c^* so $c^* = \frac{1}{2}$

2nd case q<1 , Ex >1

Here I computed the critical value c^* for c > $c^*$the process B will survive with positive probability .

we do the same computations as first case ,

C > $\frac{1}{2}$=> c >c^* so $c^* = \frac{1}{2}$

b) here I have to compute the survival probability $p_1$that the process B starting with one individual will not die out as a function of C

$q = \sum_{i=0}^{2} p_i q^i$ we assume δ is our surviving probability , δ = 1 - q

q = $\frac{1}{3} q^0 + (1-c)\,\frac{2}{3} q^1 + \frac{2c}{3} q^2$

3q = 1 + 2q +2qc + 2cq^2

2cq^2 – q (2c+1) +1 = 0

$q^2 - q\,(\frac{2c+1}{2c}) + \frac{1}{2c} = 0$

$q^2 - q\,(\frac{2c+1}{2c}) + (\frac{2c+1}{4c})^2 - (\frac{2c+1}{4c})^2 + \frac{1}{2c} = 0$

$(q - \frac{2c+1}{2c})^2 = (\frac{2c+1}{4c})^2 - \frac{1}{2c}$

$q - \frac{2c+1}{4c} = \pm\sqrt{(\frac{2c+1}{4c})^2 - \frac{1}{2c}}$

$q_1 = +\sqrt{(\frac{2c+1}{4c})^2 - \frac{1}{2c}} + \frac{2c+1}{4c}$

$q_2 = -\sqrt{(\frac{2c+1}{4c})^2 - \frac{1}{2c}} + \frac{2c+1}{4c}$

- $q_1 = \sqrt{\frac{4c^2+1+4c}{16c^2} - \frac{8c}{16c^2}} + \frac{2c+1}{4c}$

  $q_1 = \sqrt{\frac{4c^2+1-4c}{16c^2}} + \frac{2c+1}{4c}$

  $q_1 = \sqrt{(\frac{2c-1}{4c})^2} + \frac{2c+1}{4c}$

  $q_1 = \frac{2c-1}{4c} + \frac{2c+1}{4c}$

  $q_1 = \frac{4c}{4c} = 1$

  $q_1 = 1$ it's always a solution

- $q_2 = -\sqrt{(\frac{2c+1}{4c})^2 - \frac{1}{2c}} + \frac{2c+1}{4c}$

  $q_2 = -\frac{2c+1}{4c} + \frac{2c+1}{4c} = \frac{2}{4c} = \frac{1}{2c}$

our surviving probability $\delta = 1 - q_2 = 1 - \frac{1}{2c}$

c) compute the survival probability P_{10} that the process B starting with 10 individuals will not die out :

P (1 ∩ 2 ∩ 3 ∩ 4 ∩ 5 ∩ 6 ∩ 7 ∩ 8 ∩ 9 ∩ 10)

$q = (\sum_{i=0}^{2} p_i q^i)^{10}$ $\delta = 1 - q$

$\delta = (1 - \sum_{i=0}^{2} p_i q^i)^{10}$