10. Let H be a subgroup of S_n where $n > 4$, and N is a normal subgroup of H.

(a) Suppose H contains all 3-cycles of S_n. Prove that N also contains all 3-cycles of S_n.

(b) A group G is called a *solvable* group if there exists a sequences of subgroups

$$G = G_0 \geq G_1 \geq G_2 \geq \cdots \geq G_{m-1} \geq G_m = \{e\}$$

abelian

such that H_i is a normal subgroup of G_{i-1} and G_{i-1}/G_i is an abelian group for $i = 1, \ldots, m$. Prove that S_n is not a solvable group.

(c) Prove that S_1, S_2, S_3, S_4 are solvable groups.