1.) The F.T. of

$$f(x,y) = \cos(2\pi(ax+by))$$

is given as:

$$F(k_x, k_y) = \frac{1}{2}\delta(k_x - a)\,\delta(k_y - b) + \frac{1}{2}\delta(k_x + a)\,\delta(k_y + b)$$

For our function:

$$f(x,y) = \cos(2\pi(2x+y))$$

We see that $a = 2$ &
$b = 1$

Therefore the F.T. is:

$$F(k_x, k_y) = \frac{1}{2}\delta(k_x - 2)\,\delta(k_y - 1) + \frac{1}{2}\delta(k_x + 2)\,\delta(k_y + 1)$$

1.) Now lets convert $F(k_x, k_y)$ to $F(k_r, \phi)$. We can do this knowing that:

$$k_x = k_r \cos\phi, \text{ and}$$

$$k_y = k_r \sin\phi$$

for $\phi = 45°$, this gives us:

$$k_x = k_y = k_r \cdot \frac{\sqrt{2}}{2}$$

$$\therefore F(k_r, \phi) = \frac{1}{2}\,\delta\left(\frac{\sqrt{2}}{2}k_r - 2\right)\delta\left(\frac{\sqrt{2}}{2}k_r - 1\right) + \frac{1}{2}\,\delta\left(\frac{\sqrt{2}}{2}k_r + 2\right)\delta\left(\frac{\sqrt{2}}{2}k_r + 1\right)$$

$$\cancel{F = \frac{1}{2}\left\{\delta\left(\frac{1}{2}k_r^2 - \frac{3\sqrt{2}}{2}k_r + 2\right) + \frac{1}{2}k_r^2 + \frac{3\sqrt{2}}{2}k_r + 2\right)\right\}}$$

$$\cancel{= \frac{1}{2}\left[\delta\left(k_r^2 + 4\right)\right]}$$

The projection $p(r,\phi)$ will be the inverse F.T. of $F(k_r,\phi)$

$$p(r,\phi) = FT^{-1}\left\{ \frac{1}{2}\,\delta\!\left(\frac{\sqrt{2}}{2}k_r - 2\right)\delta\!\left(\frac{\sqrt{2}}{2}k_r - 1\right) + \frac{1}{2}\,\delta\!\left(\frac{\sqrt{2}}{2}k_r + 2\right)\delta\!\left(\frac{\sqrt{2}}{2}k_r + 1\right)\right\}$$