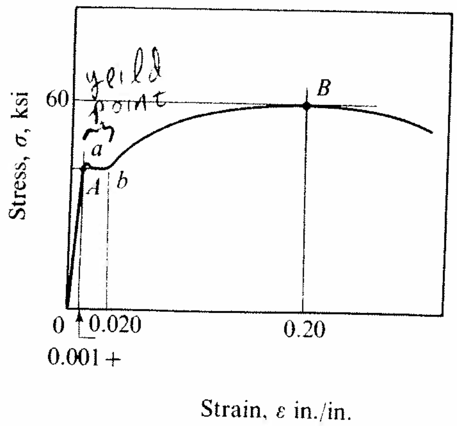

Fig. 2-3. Stress-strain diagram for mild steel

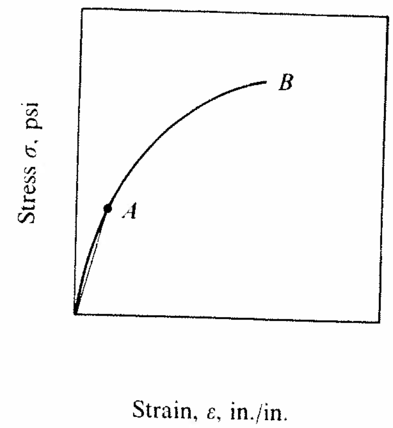

Fig. 2-4. Stress-strain diagram for a brittle material

can be recognized. One type is shown in Fig. 2-3, which is for mild steel, a *ductile* material widely used in construction. The other type is shown in Fig. 2-4. Such diverse materials as tool steel, concrete, copper, etc., have curves of this variety, although the extreme value of strain that these materials can withstand *differs drastically*. The "steepness" of these curves varies considerably. Numerically speaking, each material has its own curve. The terminal point on a stress-strain diagram represents the complete failure (rupture) of a specimen. Materials capable of withstanding large strains are referred to as *ductile materials*. The converse applies to *brittle materials*.

2-4. HOOKE'S LAW

Fortunately, one feature of stress-strain diagrams is applicable with sufficient accuracy to nearly all materials. It is a fact that for a *certain distance* from the origin the experimental values of stress vs. strain lie essentially on a straight line. This holds true almost without reservations for glass. It is true for mild steel up to some point, as A in Fig. 2-3. It holds nearly true up to very close to the failure point for many high-grade alloy steels. On the other hand, the straight part of the curve hardly exists in concrete, annealed copper, or cast iron. Nevertheless, for all practical purposes, up to some such point as A (also in Fig. 2-4), *the relationship between stress and strain may be said to be linear for all materials*. This sweeping idealization and generalization applicable to all materials became known as *Hooke's law*.* Symbolically, this law can be expressed by the equation

E = elastic modulus or Young's Modulus.

$$\sigma = E\varepsilon \quad \text{or} \quad E = \frac{\sigma}{\varepsilon} \tag{2-2}$$

These equations are usable only within an allowable limit.

*Actually Robert Hooke, an English scientist, worked with springs and not with rods. In 1676 he announced an anagram "c e i i i n o s s s t t u v," which in Latin is *Ut Tensio sic Vis* (the force varies as the stretch).

which simply means that stress is directly proportional to strain where the constant of proportionality is E. This constant E is called the *elastic modulus*, modulus of elasticity, or Young's modulus.* As ε is dimensionless, E has the units of stress in this relation. In the English system of units it is usually measured in pounds per square inch, while in the SI units it is measured in newtons per square meter (or pascals).

Graphically E is interpreted as the slope of a straight line from the origin to the rather vague point A on a stress-strain diagram. The stress corresponding to the latter point is termed the *proportional limit* of the material. Physically the elastic modulus represents the stiffness of the material to an imposed load. *The value of the elastic modulus is a definite property of a material.* From experiments it is known that ε is *always a very small quantity*, hence E must be a large one. Its approximate values are tabulated for a few materials in Table 1 of the Appendix. For most steels, E is between 29 and 30×10^6 psi, or 200 and 207×10^9 N/m^2.

It follows from the foregoing discussion that *Hooke's law applies only up to the proportional limit of the material.* This is highly significant as in most of the subsequent treatment the derived formulas are based on this law. Clearly then, such formulas will be limited to the material's behavior in the lower range of stresses.

Some materials, notably single crystals, possess different elastic moduli in different directions with reference to their crystallographic planes. Such materials, having different physical properties in different directions, are termed nonisotropic. A consideration of such materials is *excluded* from this text. The vast majority of engineering materials consist of a large number of *randomly* oriented crystals. Because of this random orientation of crystals, properties of materials become essentially alike in any direction.† Such materials are called *isotropic. Throughout this text complete homogeneity* (sameness) *and isotropy of materials is assumed.*

2-5. FURTHER REMARKS ON STRESS-STRAIN DIAGRAMS

In addition to the proportional limit defined in Art. 2-4, several other interesting points can be observed on the stress-strain diagrams. For instance, the highest points (B in Figs. 2-3 and 2-4) correspond to the *ultimate* strength of a material. *Stress* associated with the remarkably long plateau ab in Fig. 2-3 is termed the *yield point* of a material. As will be brought out later, this remarkable property of mild steel, in common with other *ductile* materials, is significant in stress analysis. For the present, note that at an essentially constant stress, strains 15 to 20 times those that take place up to the proportional limit occur during yielding. At the yield point a large amount of

*Young's modulus is so called in honor of Thomas Young, an English scientist. His *Lectures on Natural Philosophy*, published in 1807, contain a definition of the modulus of elasticity.

†Rolling operations produce preferential orientation of crystalline grains in some materials.