$11 \equiv 1 \pmod{10}$

$11 \equiv 2 \pmod{3}$

MATH 3140 Due in class, Tuesday, April 2, 2019

Homework 9

Problems:

(1) 7.23 (3pts.)

(2) 7.28 (3pts.) — $5-3+5-8+0-1 \to 1-0+8-5+3-5$

(3) (4pts.) Define the relation $\sim$ on $\mathbb{R}$ as follows: for $x, y \in \mathbb{R}$, $x \sim y$ if and only if $x-y \in \mathbb{Q}$.

(a) Prove that $\sim$ is an equivalence relation on $\mathbb{R}$.

(b) List four different real numbers that are in the equivalence class of $\sqrt{2}$.

(c) If $a \in \mathbb{Q}$, what is the equivalence class of a?

$n = \{d_k, d_{k-1}, \ldots, d_1, d_0\}_{10}$ $\quad 11 \mid n$ iff $11 \mid \sum_{i=0}^{k} d_i \quad 0 \le i \le k$

$= d_0 + d_1 10^1 + d_2 10^2 + \ldots + d_k 10^k$

$0, 1, 2, 3, \ldots, 9$

$11, 22, 33, 44, \ldots, 99$ — $10 \equiv -1 \pmod{11}$

~~$101, 111, 121, 131, \ldots, 191$~~ $\equiv \pmod{11}$

$1001, 1111, 1221, 1331, \ldots, 1991$ — $\equiv \pmod{11}$

$2002, 2112, 2222, 2332, \ldots, 2992$ —

3993 —

$\vdots$

9999 —

10901

$10001, 10101, 10201, \ldots$

$= d_0 - d_1 (10)^1 \pmod{11} + d_2 (10)^2 \pmod{11} - d_3 (10)^3 \pmod{11} + \ldots + d_k 10^k \pmod{11}$

$= d_0 - d_1 + d_2 - d_3 + \ldots + d_k (-1)^k \pmod{11}$