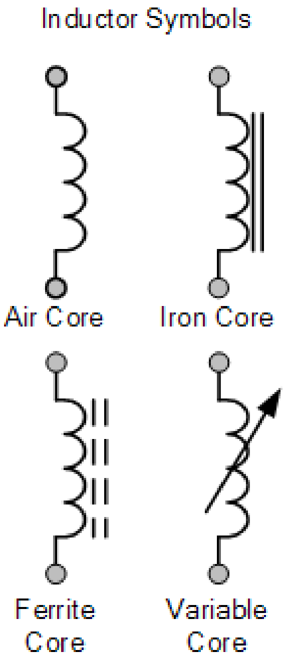
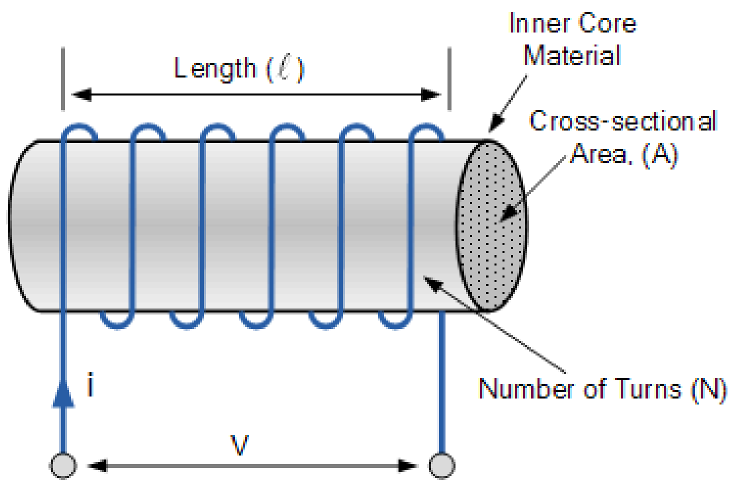
Inductor

2017년 2월 28일 화요일   오전 11:56





Inductor is an electrical component storing an electrical energy in form of a magnetic field $\vec{B}$. When current *i* flows to the inductor, the magnetic field forms within the magnetic core surrounded by a conducting coil carrying *i*. If *i(t)* is time-varying, then $\vec{B}(t)$ is also time-varying. Then Faraday's law of induction is applied; The equation of Faraday's law of induction for a closed loop is

$\varepsilon = \oint_C \vec{E}_{emf} \cdot d\vec{l} = -\frac{d\Phi}{dt}$, where ε is EMF (ElectroMotive Force) on the loop C and $\Phi = \int \vec{B} \cdot d\vec{S}$ is a magnetic flux through the core.

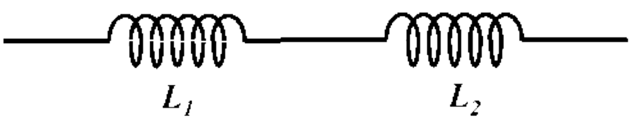
If the core is N-multiply and closed wound by the coil, then the coil can be thought as identical simple closed loops in a series connection. Each loop is applied by $\varepsilon = \oint_C \vec{E}_{emf} \cdot d\vec{l} = -\frac{d\Phi}{dt}$. So, a EMP on the coil is equal to N times each ε or $\varepsilon_{coil} = \int_{coil} \vec{E}_{emf} \cdot d\vec{l} \cong -N\frac{d\Phi}{dt}$, where the line-integral is done along the coil.

Since the current *i(t)* flows on the coil, a current density $\vec{J} = \sigma\vec{E}$ where σ is a conductivity of a material of the coil exists. It should be noted that $\vec{E}$ in $\vec{J} = \sigma\vec{E}$ is not necessarily equal to $\vec{E}_{emf}$; $\vec{E}$ is a sum of all electric field on the coil. $\vec{E}$ is generally a sum of an electrostatic field $\vec{E}_{st}$ and EMF-field $\vec{E}_{emf}$; $\vec{E} = \vec{E}_{st} + \vec{E}_{emf}$. $\vec{E}$ is not conservative $\left(\nabla \times \vec{E} \neq 0\right)$, means that the line-integral of it is integral-path-dependent, so a potential difference over the inductor is hardly defined. However, we can define an effective potential difference or a voltage $V = -\int_{coil} \vec{E} \cdot d\vec{l}$ as the line-integral-path as the coil is singly given. $\int_{coil} \vec{E} \cdot d\vec{l} = \int_{coil} \left(\vec{E}_{st} + \vec{E}_{emf}\right) \cdot d\vec{l} \cong \int_{coil} \vec{E}_{emf} \cdot d\vec{l}$ as the line-integral of $\vec{E}_{st}$ along the closed loop is always zero, so $V \cong -\int_{coil} \vec{E}_{emf} \cdot d\vec{l} \cong N\frac{d\Phi}{dt}$. Yes, the volage on the inductor is equal to a negative EMP on the coil of the inductor.

The inductance of the inductor is defined as $\mathrm{L} = \frac{N\Phi}{i}$; L is a kind of a proportional constant of the magnetic flux Φ to the current i. [1] With L, $V \cong N\frac{d\Phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$. This is a basic equation of the inductor.

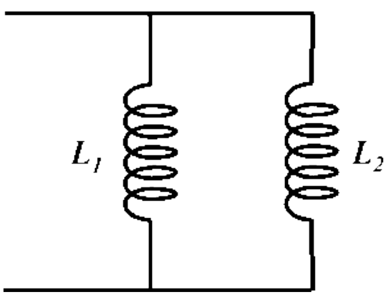
The inverse Fourier transformation of the equation is [2]; $V(t) \cong L\frac{di(t)}{dt} \to \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} V(\omega)e^{j\omega t}d\omega = L\frac{d}{dt}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} i(\omega)e^{j\omega t}d\omega = L\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} i(\omega)\frac{de^{j\omega t}}{dt}d\omega = L\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} j\omega i(\omega)e^{j\omega t}d\omega \to V(\omega) \cong j(\omega L)i(\omega)$, where ω is an angular frequency. Here, the Fourier basis is chosen $e^{i\omega t}$ instead of $e^{-i\omega t}$, as it is a conventional choice in circuit analysis. $\tilde{Z} = j\omega L$ is an impedance of the inductor in the generalized Ohm's law $\tilde{V} = \tilde{\imath}\tilde{Z}$ where cap ~ tells that a quantity with it is in a complex number representation.

Likely to resistors, the series connection of inductances gives an equivalent impedance is



$\tilde{Z}_{eq} = (j\omega L_1) + (j\omega L_2) = j\omega(L_1 + L_2) = j\omega L_{eq} \to L_{eq} = L_1 + L_2$.

For parallel connection,



$$\tilde{Z}_{eq} = \left(\frac{1}{j\omega L_1} + \frac{1}{j\omega L_2}\right)^{-1} = \left(\frac{j\omega L_1 + j\omega L_2}{-\omega^2 L_1 L_2}\right)^{-1} = -\frac{\omega L_1 L_2}{jL_1 + jL_2} = j\omega\frac{L_1 L_2}{L_1 + L_2} = j\omega L_{eq} \to L_{eq} = \frac{L_1 L_2}{L_1 + L_2}.$$

Reference

1. Matthew N. O. Sadiku, *Elements of Electromagnetics*, 3rd ed. Oxford University Press, USA (1000), 2000.
2. G. Arfken, *Mathematical Methods for Physicists 6th*, vol. 40, no. 4. 2005.