PHY 5340

Homework #2 Solutions

1.

∇2□=1□2∂2□∂□2

sin□·□−□□□·□−□□=sin□□□−□□□□□−□□

∇2□=∂2∂□2sin□□□−□□□□□−□□=∂∂□□sin□□□−□□−□□□−□□cos□□□−□□□□□−□□2
=−□□2sin□□□−□□□□□−□□+2□□2sin□□□−□□□□□−□□3−2□□2cos□□□−□□□□□−□□2

∂2□∂□2=∂2∂□2sin□□□−□□□□□−□□=∂∂□□sin□□□−□□−□□□−□□cos□□□−□□□□□−□□2
=−□2sin□□□−□□□□□−□□+2□2sin□□□−□□□□□−□□3−2□2cos□□□−□□□□□−□□2

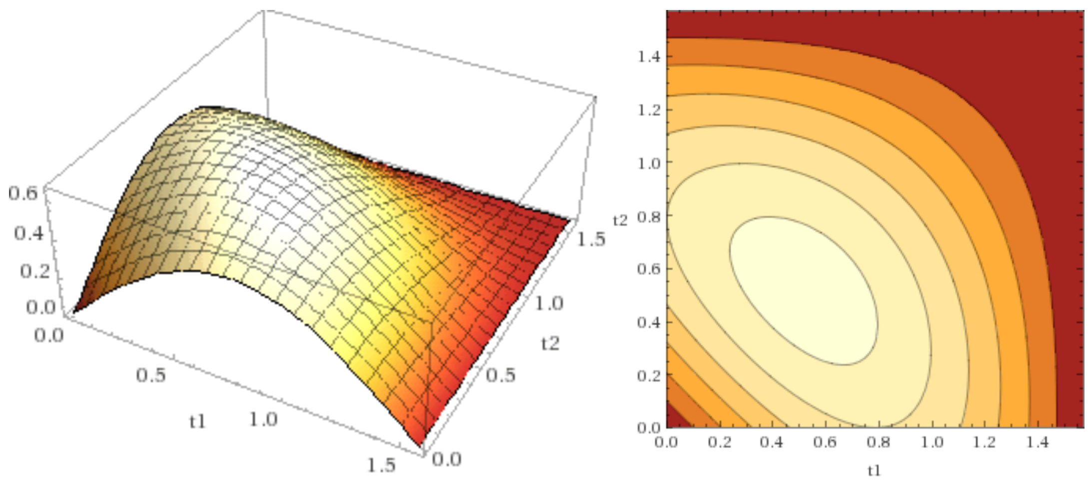
1□2∂2□∂□2=□□2□2∂2□∂□2=−□□2sin□□□−□□□□□−□□+2□□2sin□□□−□□□□□−□□3−2□□2cos□□□
−□□□□□−□□2

2. If the three polarizers each rotate the light by angles of θ_1, θ_2, and θ_3, then the total light transmitted with be

$E_{final} = E_{initial}*\cos(\theta_1)*\cos(\theta_2)*\cos(\theta_3)$.

Using 3 polarizers to rotate the light by 90˚, the most light will get through if each polarizer rotates the light by 30˚.

Knowing that $\theta_1+\theta_2+\theta_3 = 90˚$, plot $f(\theta_1,\theta_2) = \cos(\theta_1)\cos(\theta_2)\cos(90˚-\theta_1-\theta_2)$ to see that it is maximum at $\theta_1 = \theta_2 = \theta_3$.



It'd be better to go on and show that the normal to the tangent plane is vertical at $\theta_1 = \theta_2 = \theta_3$, but that's not necessary at this time.

$E_{final} = E_{initial}*\cos(\theta_1)*\cos(\theta_2)*\cos(\theta_3)$
$= E_{initial}*\cos^3(\pi/6)$
$= E_{initial} * 0.649$

Remembering that I is proportional to E^2, $I_{final} = I_{initial}*(0.649)^2 = I_{initial} * 0.422$.

64.9% of the incident Electric field magnitude and 42.2% of the incident light intensity can be transmitted through 3 linear polarizers with the final polarization rotated 90˚ from the initial polarization.

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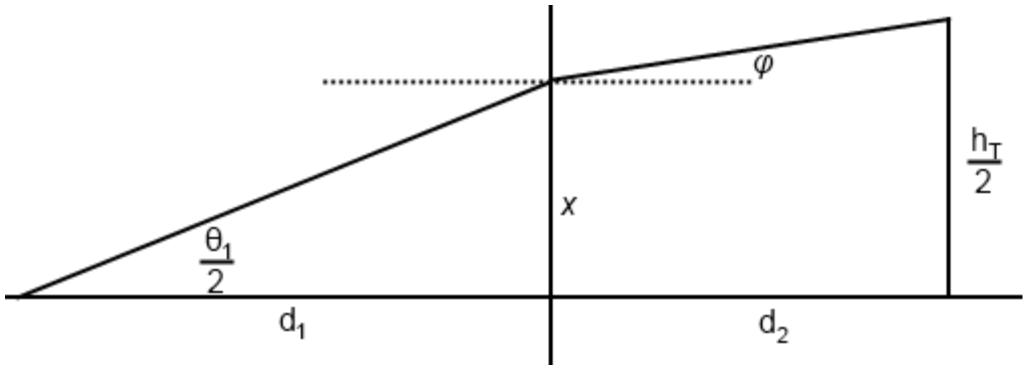
3. $I = I_0 \cos^2\theta$

Since the light is unpolarized, all angles of polarization are equally populated. Averaging over the angles from 0 to 90˚ and calculating the amount of light that gets through at each polarization yields
$I = I_0 \frac{1}{\pi/2}\int_0^{\pi/2}\cos^2\theta\, d\theta = I_0 \frac{1}{\pi/2}\frac{\pi}{4} = \frac{I_0}{2}$

4.

tan□02≈□02=ℎ□2□1+□2   =>   □0=ℎ□□1+□2



tan□12=□□1   ;   tan□=ℎ□2−□□2   ;   □1sin□12=□2sin□

=>

□12≈□□1   ;   □≈ℎ□2−□□2   ;   □1□12≈□2□

□1=ℎ□□11+□2□1□1□2

□0□1=□1+□2□1□2□1+□2

=>

□0□1=3+10011.333+100=0.759

To test the amount that the small angle approximation was inaccurate, let us notice that we commonly approximated sin(θ) = θ and tan(θ) = θ. Plugging in the numbers given, $\theta_1 \approx 3.6^\circ$.

$\sin(\theta_1)/\theta_1 = 0.999$
$\tan(\theta_1)/\theta_1 = 1.001$

Thus, the small angle approximation is approximately off by 0.1%. Obviously, it would be better to fully solve this problem without the small angle approximation and compare the results for $\theta_1$from both methods, but that's not necessary for this quick test.

5.

sin□1=□□sin□2
□□sin□3=sin□4
□2+□3=60°

Red:

$\sin 60° = 1.45 \sin\theta_2 \quad \Rightarrow \quad \theta_2 = 36.67°$
$\theta_2 + \theta_3 = 60° \quad \Rightarrow \quad \theta_3 = 23.34°$
$1.45 \sin\theta_3 = \sin\theta_4 \quad \Rightarrow \quad \theta_4 = 35.04°$

Blue:
$\sin 60° = 1.48 \sin\theta_2 \quad \Rightarrow \quad \theta_2 = 35.81°$
$\theta_2 + \theta_3 = 60° \quad \Rightarrow \quad \theta_3 = 24.19°$
$1.48 \sin\theta_3 = \sin\theta_4 \quad \Rightarrow \quad \theta_4 = 37.33°$

$\Delta\theta = 2.29°$

6.

Critical angle:
$1.5 \sin\theta_c = 1.33 \quad \Rightarrow \quad \theta_c = 62.46°$

Minimum input angle:
$1.45 \sin\theta_i = 1.5 \sin\theta_c \quad \Rightarrow \quad \theta_i = 66.53°$

So any light incident upon the glass at an angle above 66.53˚ will be completed reflected off of the glass-water interface.