An alternate proof of the $e - \delta$ identity is to consider the determinant

$$\begin{vmatrix} \delta_1^1 & \delta_2^1 & \delta_3^1 \\ \delta_1^2 & \delta_2^2 & \delta_3^2 \\ \delta_1^3 & \delta_2^3 & \delta_3^3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1.$$

By performing a permutation of the rows of this matrix we can use the permutation symbol and write

$$\begin{vmatrix} \delta_1^i & \delta_2^i & \delta_3^i \\ \delta_1^j & \delta_2^j & \delta_3^j \\ \delta_1^k & \delta_2^k & \delta_3^k \end{vmatrix} = e^{ijk}.$$

By performing a permutation of the columns, we can write

$$\begin{vmatrix} \delta_r^i & \delta_s^i & \delta_t^i \\ \delta_r^j & \delta_s^j & \delta_t^j \\ \delta_r^k & \delta_s^k & \delta_t^k \end{vmatrix} = e^{ijk} e_{rst}.$$

Now perform a contraction on the indices i and r to obtain

$$\begin{vmatrix} \delta_i^i & \delta_s^i & \delta_t^i \\ \delta_i^j & \delta_s^j & \delta_t^j \\ \delta_i^k & \delta_s^k & \delta_t^k \end{vmatrix} = e^{ijk} e_{ist}.$$

Summing on i we have $\delta_i^i = \delta_1^1 + \delta_2^2 + \delta_3^3 = 3$ and expand the determinant to obtain the desired result

$$\delta_s^j \delta_t^k - \delta_t^j \delta_s^k = e^{ijk} e_{ist}.$$