same frequency and wavevector but with a random distribution of phase angles. Prove that the beam is first-order coherent at any pair of space–time points.

Problem 3.2 Consider the beam of light produced by excitation of two stable waves, where the electric field is

$$E(z,t) = E_1 \exp(ik_1 z - i\omega_1 t) + E_2 \exp(ik_2 z - i\omega_2 t). \tag{3.4.23}$$

Prove that the light is first-order coherent at all pairs of points.

Problem 3.3 Consider a beam of light produced by excitation of two waves as in eqn (3.4.23), but where both exhibit random amplitudes and phases. If the average intensity is equally divided between the two waves, prove that

$$\left|g^{(1)}(\tau)\right| = \left|\cos\left\{\tfrac{1}{2}(\omega_1 - \omega_2)\tau\right\}\right|. \tag{3.4.24}$$

Note that the property in eqn (3.4.19) does not apply to any of the field excitations considered in these three problems.

Problem 3.4 Consider light from a source that simultaneously has collision and Doppler broadening. Prove that the degree of first-order coherence is

$$g^{(1)}(\tau) = \exp\left\{-i\omega_0\tau - \gamma_{\mathrm{coll}}|\tau| - \tfrac{1}{2}\Delta^2\tau^2\right\}. \tag{3.4.25}$$

3.5 Interference fringes and frequency spectra

With the properties of the degree of first-order coherence now evaluated, we may return to the theory of the Mach–Zehnder interferometer outlined in §3.3. The output intensity, given by eqns (3.3.9) and (3.3.10), can be written in the more explicit form

$$\langle \bar{I}_4(t)\rangle = 2|\mathcal{R}|^2|\mathcal{T}|^2\langle \bar{I}(t)\rangle\left\{1 + \exp\left[-|z_1 - z_2|/c\tau_c\right]\cos\left[\omega_0(z_1 - z_2)/c\right]\right\}. \tag{3.5.1}$$

light is assumed to have Lorentzian broadening with a degree of first-