TT Ball Trajectory

$dia := 0.041$		Diameter of a ping pong ball in meters
$r := \frac{dia}{2}$	$r = 0.0205$	Radius of a ping pong ball in meters
$A := \pi \cdot r^2$	$A = 0.00132$	Area of cross section of a ping pong ball in m^2.
$mass := 0.0027$		Mass of a ping pong ball in kg
$I := \frac{2 \cdot mass \cdot r^2}{3}$	$I = 7.5645 \times 10^{-7}$	TT ball inertia $Kg*m^2$
$\rho := 1.225$		Density of dry air in kg/m^3 at sea level
$C_d := 0.5$		Ideal drag coefficient for a sphere. Unitless
$C_m := 0.29$		Magnus coefficient. Unitless
$\eta := 1.78 \cdot 10^{-5}$		Viscosity of air in $\frac{kg}{m \cdot s}$
$b := 6 \cdot \pi \cdot \eta \cdot r$	$b = 6.878203 \times 10^{-6}$	Viscous friction coefficient.

Magnus combined constant

$S := \frac{4}{3} \cdot \left(4 \cdot \pi^2 \cdot r^3 \cdot \rho\right)$	$S = 0.000556$	From a NASA listed below
$v_0 = 17$		Initial velocity in m/s
$\theta = 14\,deg$	$\theta = 0.244346$	Angle relative to horizontal
$rps_0 = 25$		Initial spin in revolutions per second

$$Q := \begin{pmatrix} x \leftarrow -1.5 \\ x' \leftarrow v_0 \cdot \cos(\theta) \\ y \leftarrow 0.05 \\ y' \leftarrow v_0 \cdot \sin(\theta) \\ rps \leftarrow rps_0 \end{pmatrix}$$

Q is the state.
Initial horizontal and vertical positions and velocities and spin

$$D(t, Q) := \left| \begin{array}{l} \begin{pmatrix} x \\ x' \\ y \\ y' \\ rps \end{pmatrix} \leftarrow Q \\ x'' \leftarrow -\dfrac{b \cdot x'}{mass} - \operatorname{sign}(x') \cdot \dfrac{\rho \cdot C_d \cdot A \cdot \dfrac{x'^2}{2}}{mass} + \dfrac{S \cdot y' \cdot rps}{mass} \\ y'' \leftarrow -9.8 - \dfrac{b \cdot y'}{mass} - \operatorname{sign}(y') \cdot \dfrac{\rho \cdot C_d \cdot A \cdot \dfrac{y'^2}{2}}{mass} - \dfrac{S \cdot x' \cdot rps}{mass} \\ rps' \leftarrow -0.03 \cdot rps \\ \begin{pmatrix} x' \\ x'' \\ y' \\ y'' \\ rps' \end{pmatrix} \end{array} \right.$$

Break out the state variables into horizontal position, horizontal velocity, vertical position, vertical velocity and radians per second

Compute horizontal deceleration

Computes vertical acceleration/deceleration

Slow down the spin of the ball exponentially. Slow down 3% per second. This seems slow but I have seen ball spin for many seconds.

Return rate of change in the state

TT Ball Trajectory

$SimTime := 1$ — Simulation time

$\Delta t := 0.001$ — Time increment

$N := \frac{SimTime}{\Delta t}$ $\quad N = 1000$ — Number of time increments

$Z := rkfixed(Q, 0, SimTime, N, D)$ — Integrate using Runge-Kutta

	Time	x	x'	y	y'	rps
	0	1	2	3	4	5
0	0	-1.5	16.495027	0.05	4.112672	25
1	0.001	-1.483515	16.475193	0.054064	4.015596	24.99925
2	0.002	-1.46705	16.454958	0.058031	3.918741	24.9985
3	0.003	-1.450605	16.434326	0.061902	3.822109	24.99775
4	0.004	-1.434181	16.4133	0.065676	3.725696	24.997
5	0.005	-1.417779	16.391882	0.069353	3.629503	24.99625
6	0.006	-1.401398	16.370075	0.072935	3.533529	24.9955
7	0.007	-1.385039	16.347883	0.07642	3.437772	24.994751
8	0.008	-1.368702	16.325308	0.07981	3.342231	24.994001
9	0.009	-1.352388	16.302353	0.083105	3.246905	24.993251
10	0.01	-1.336097	16.279021	0.086304	3.151794	24.992501
11	0.011	-1.31983	16.255315	0.089408	3.056896	24.991751
12	0.012	-1.303587	16.231237	0.092418	2.962211	24.991002
13	0.013	-1.287368	16.206791	0.095333	2.867737	24.990252
14	0.014	-1.271173	16.181978	0.098154	2.773473	24.989502
15	0.015	-1.255004	16.156802	0.10088	2.679419	24.988753

$Z =$ (table above)

$t := Z^{\langle 0 \rangle}$ $\quad x := Z^{\langle 1 \rangle}$ $\quad x' := Z^{\langle 2 \rangle}$ $\quad y := Z^{\langle 3 \rangle}$ $\quad y' := Z^{\langle 4 \rangle}$ $\quad rps := Z^{\langle 5 \rangle}$

$$i := \left| \begin{array}{l} i \leftarrow 0 \\ \text{while } y_i > 0 \\ \quad i \leftarrow i + 1 \end{array} \right.$$

$i = 103$

Find the point where the center of the ball goes below table top.

$n := 0..i$

$x := submatrix(x, 0, i, 0, 0)$ — extract the horizontal positions from Z

$y := submatrix(y, 0, i, 0, 0)$ — extract the vertical positions from Z

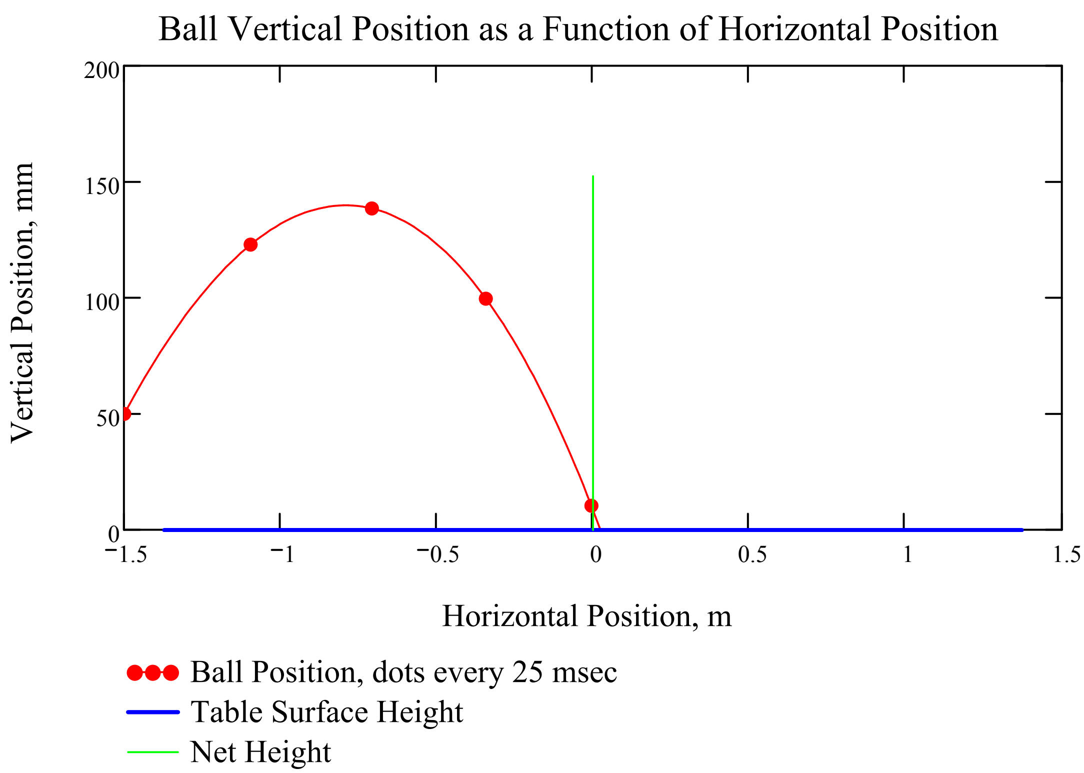
$table_n := 0$ — table height is 0

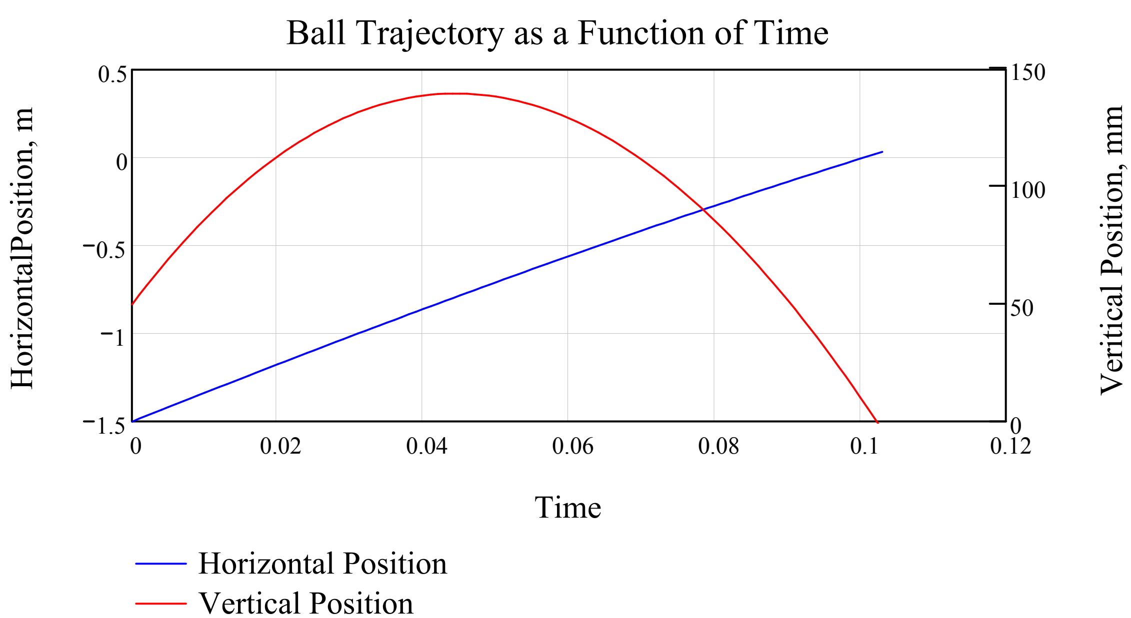
$net_n := 0.1525$ — Net height in meters

$tbl\_len := 9 \cdot ft$ $\quad tbl\_len = 2.7432\,m$ — Table length

$tbl\_end := \frac{tbl\_len}{2}$ $\quad tbl\_end = 1.3716\,m$ — Distance from net. + for the far end. - for the near end

TT Ball Trajectory





$v_0 \equiv 17$ $\theta \equiv 14 \cdot deg$ $rps_0 \equiv 25$ $i \cdot \Delta t = 0.103$ seconds

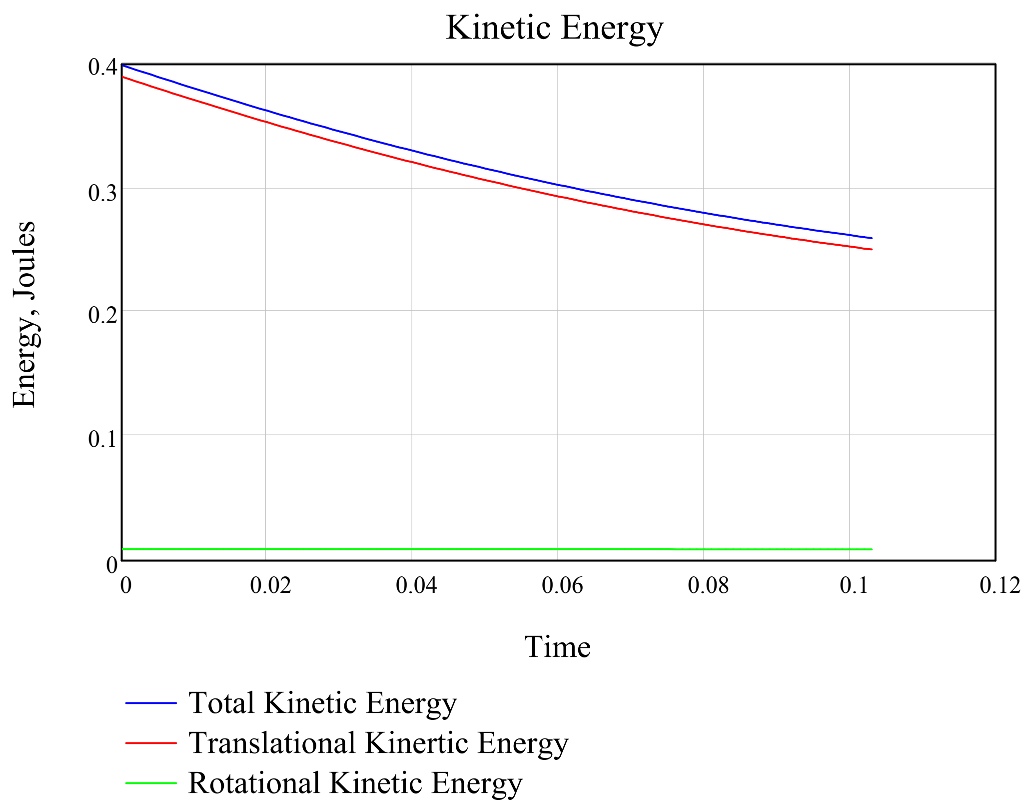
Kinetic Energy

$$Er_n := \frac{1}{2} \cdot I \cdot \left[(2 \cdot \pi \cdot rps)_n \right]^2$$

Rotational Kinetic Energy
I is the rotational inertia and
2*π*rps_n angular velocity in
rad/s

$$Et_n := \frac{1}{2} \cdot mass \cdot \left[\left(x'_n\right)^2 + \left(y'_n\right)^2 \right]$$

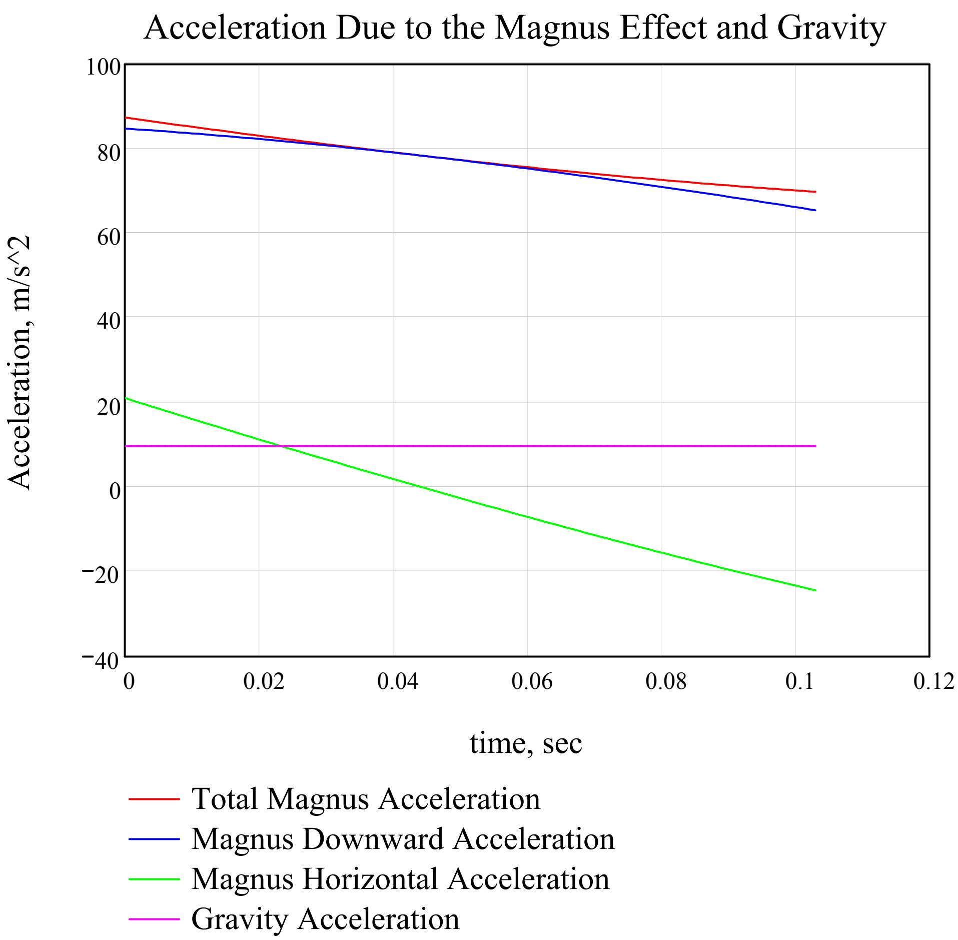
Translational Kinetic Energy
x` is the horizontal velocity
y` is the vertical velocity



Accelerations Due to the Magnus Effect

$$x''_n := \frac{S \cdot y'_n \cdot rps_n}{mass}$$ Horizontal acceleration due to the Magnus effect

$$y''_n := \frac{S \cdot x'_n \cdot rps_n}{mass}$$ Vertical acceleration due to the Magnus effect



Verify Units

add units to the variables above.

$dia := dia \cdot m$		TT ball diameter
$r := \frac{dia}{2}$	$r = 0.0205\,m$	TT ball radius
$A := \pi \cdot r^2$	$A = 0.00132\,m^2$	TT ball cross sectional area
$mass := mass \cdot kg$		TT ball mass
$I := \frac{2 \cdot mass \cdot r^2}{3}$	$I = 7.5645 \times 10^{-7}\,m^2 \cdot kg$	TT ball inertia
$\rho := 1.225 \cdot \frac{kg}{m^3}$		Density of air
$\eta := 1.78 \cdot 10^{-5} \cdot \frac{kg}{m \cdot s}$		Viscous friction coefficient
$b := 6 \cdot \pi \cdot \eta \cdot r$	$b = 6.878203 \times 10^{-6}\,\frac{kg}{s}$	Combine viscous friction coefficient
$S := \frac{4}{3} \cdot \left(4 \cdot \pi^2 \cdot r^3 \cdot \rho\right)$	$S = 0.000556\,kg$	Combined Magus coefficient
$v_0 := v_0 \cdot \frac{m}{s}$		Initial velocity, combined x and y
$\theta = 0.244346$	$\theta = 0.244346$	Trajectory angle about horizontal in radians.
$rps_0 := rps_0 \cdot \frac{rev}{sec}$		Initial spin
$x' := v_0 \cdot \cos(\theta)$	$x' = 16.495027\,\frac{m}{s}$	Initial horizontal velocity
$y' := v_0 \cdot \sin(\theta)$	$y' = 4.112672\,\frac{m}{s}$	Initial vertical velocity

Acceleration and Units

All the terms result in an acceleration in meters per second squared to the units are consistent.

Horizontal acceleration. In this case it is negative or decelerating.

$$x'' := -\frac{b \cdot x'}{mass} - \text{sign}(x') \cdot \frac{\rho \cdot C_d \cdot A \cdot \frac{x'^2}{2}}{mass} + \frac{S \cdot y' \cdot rps_0}{mass} \qquad x'' = 92.128544 \, \frac{m}{s^2}$$

Vertical acceleration. Negative values mean downwards

$$y'' := -g - \frac{b \cdot y'}{mass} - \text{sign}(y') \cdot \frac{\rho \cdot C_d \cdot A \cdot \frac{y'^2}{2}}{mass} - \frac{S \cdot x' \cdot rps_0}{mass} \qquad y'' = -545.445873 \, \frac{m}{s^2}$$

All the individual terms have consistent units of acceleration.

$$\frac{b \cdot x'}{mass} = 0.042021 \, \frac{m}{s^2}$$

Viscous damping

$$\text{sign}(x') \cdot \frac{\rho \cdot C_d \cdot A \cdot \frac{x'^2}{2}}{mass} = 40.745158 \, \frac{m}{s^2}$$

Drag

$$\frac{\frac{4}{3} \cdot \left(4 \cdot \pi^2 \cdot r^3 \cdot \rho\right) \cdot rps_0 \cdot v_0}{mass} = 549.415844 \, \frac{m}{s^2}$$

Magus effect

NASA Glenn Research Center

Ideal Lift of a Spinning Ball
https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/ideal-lift-of-a-spinning-ball/

$$L = \text{Lift} = \frac{4}{3}\cdot\left(4\cdot\pi^2\cdot b^3\cdot s\cdot\rho\cdot V\right)$$

From the NASA document. Lift can also be drop. It depends on the directins of the spin

$V := v_0 \qquad V = 17\,\frac{m}{s}$

Velocity of the ball

$b := r \qquad b = 0.0205\,m$

Radius of the ball

$s := 25\cdot\frac{rev}{sec}$

Notice that the spin is in revolutions per second not radians per second

$$\frac{4}{3}\cdot\left(4\cdot\pi^2\cdot b^3\cdot s\cdot\rho\cdot V\right) = 1.483423\,N$$

This is way too high

$$\frac{\frac{4}{3}\cdot\left(4\cdot\pi^2\cdot b^3\cdot s\cdot\rho\cdot V\right)}{mass} = 549.415844\,\frac{m}{s^2}$$

This is way too high.