http://www.physicsforums.com/showthread.php?t=473308

Write the indices of the matrices in the trace out explicitly

$$\begin{aligned} f(X) &= \mathrm{Tr}(X^{\mathrm{T}} A\, X) - 2\,\mathrm{Tr}(X^{\mathrm{T}} B\, C) \\ &= X_{b a}\, A_{b c}\, X_{c a} - 2\, X_{b a}\, B_{b c}\, C_{c a}, \end{aligned}$$

where Einstein summation convention is used (sum over repeated indices).

To take the derivative with respect to X we need to know that

$$\frac{\partial X_{a b}}{\partial X_{d e}} = \delta_{d a}\, \delta_{e b} = \begin{cases} 1 & \text{if the indices match} \\ 0 & \text{else} \end{cases}$$

δ is called the Kronecker-δ symbol. So then just use linearity and product rule for the derivative to get

$$\begin{aligned} \frac{\partial f(X)}{\partial X_{d e}} &= \frac{\partial X_{b a}}{\partial X_{d e}} A_{b c}\, X_{c a} + X_{b a}\, A_{b c} \frac{\partial X_{c a}}{\partial X_{d e}} - 2 \frac{\partial X_{b a}}{\partial X_{d e}} B_{b c}\, C_{c a} \\ &= \delta_{d b}\, \delta_{e a}\, A_{b c}\, X_{c a} + X_{b a}\, A_{b c}\, \delta_{d c}\, \delta_{e a} - 2\, \delta_{d b}\, \delta_{e a}\, B_{b c}\, C_{c a} \\ &= A_{d c}\, X_{c e} + X_{b e}\, A_{b d} - 2\, B_{d c}\, C_{c e} \\ &= (A\, X)_{d e} + (A^{\mathrm{T}} X)_{d e} - 2\,(B\, C)_{d e} \end{aligned}$$

where in the last line we gathered up the indices again and hid the sums.

Finally, rewrite in matrix form

$$\frac{\partial f(X)}{\partial X} = A\, X + A^{\mathrm{T}} X - 2\, B\, C = (A + A^{\mathrm{T}})\, X - 2\, B\, C$$

So $f(X)$ is at a stationary point if

$$(A + A^{\mathrm{T}})\, X = 2\, B\, C$$

Symbolic Checks:

```
In[1]:= (n = 2; X = Array[x, {n, n}];
          A = Array[a, {n, n}]; B = Array[b, {n, n}]; C = Array[c, {n, n}]);

In[2]:= D[X, {X}] == Array[Boole[#1 == #3 && #2 == #4] &, {n, n, n, n}]

Out[2]= True

In[3]:= D[Tr[Xᵀ.A.X], {X}] == Array[D[Tr[Xᵀ.A.X], x[##]] &, {n, n}]

Out[3]= True

In[4]:= D[Tr[Xᵀ.A.X] - 2 Tr[Xᵀ.B.C], {X}] == (A + Aᵀ).X - 2 B.C // Expand

Out[4]= True
```