increases faster, like Z^2. If we regard the emission of a heavy fragment as a decay process similar to α decay, then we can regard heavy nuclei as residing very close to the top of the potential well in Figure 8.3, where the Coulomb barrier is very thin and easily penetrable. Fission can thus occur spontaneously as a natural decay process, or it can be induced through the absorption of a relatively low-energy particle, such as a neutron or a photon (producing excited states or compound-nuclear states that are high enough in energy to surmount or more easily penetrate the barrier).

Although any nucleus will fission if we provide enough excitation energy, as a practical matter the process is important only for heavy nuclei (thorium and beyond). The applicability of fission for obtaining large total energy releases was realized soon after its discovery, for another characteristic of the process is that every neutron-induced fission event produces, in addition to the two heavy fragments, several neutrons which can themselves induce new fission events. This *chain reaction* of fissions can occur very rapidly and without control, as in a fission explosive, or slowly and under careful control, as in a fission reactor. Because of these spectacular and awesome applications, nuclear fission plays a prominent role in many technical processes and in political decisions as well.

13.1 WHY NUCLEI FISSION

The energetic preference for nuclei to fission can be understood immediately from the binding energy per nucleon, Figure 3.16. A heavy nucleus in the uranium region has a binding energy of about 7.6 MeV/nucleon. If ^{238}U were to divide into two equal fragments with $A \simeq 119$, their binding energy per nucleon would be about 8.5 MeV. Going to a more tightly bound system means that energy must be released; that is, the energy changes from bound $^{238}_{92}U$ at $-238 \times 7.6 = -1809$ MeV to two bound $^{119}_{46}Pd$ nuclei at $-2 \times 119 \times 8.5 = -2033$ MeV. To conserve energy, the final state must include an extra 214 MeV, which can appear in a variety of forms (neutrons, β and γ emissions from the fragments) but which appears primarily ($\sim 80\%$) as kinetic energy of the fragments as Coulomb repulsion drives them apart. In calculating decay probabilities, there is a term that depends on the energy release—the more energy is released, the more ways there are for the decay products to share the energy, the greater the number of final states to decay into, and the higher the decay probability. With such a large energy release, fission ought to be a decay means that is readily available for these nuclei as they "climb the curve of binding energy."

While the fission decay mode does indeed exist, it is not nearly so probable as our discussion might indicate—it does not compete successfully with the spontaneous α decay of ^{238}U ($t_{1/2} = 4.5 \times 10^9$ y, while the partial half-life for fission is about 10^{16} y), and it does not become an important decay process until we get to nuclei of mass 250 and above. What inhibits the fission process is the Coulomb barrier, which also inhibits the analogous α-decay process. If we divide ^{238}U into two identical fragments that are just touching at their surfaces (separation = R_1

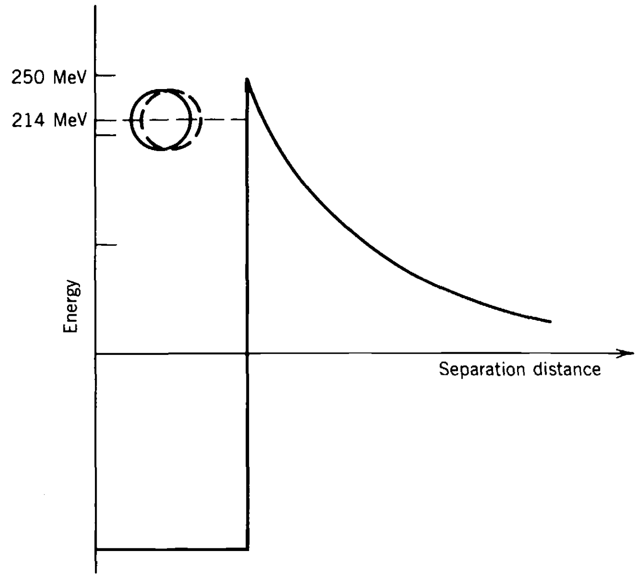

Figure 13.1 Inside its nuclear potential well, ^{238}U may perhaps exist instantaneously as two fragments of ^{119}Pd, but the Coulomb barrier prevents them from separating.

$+ R_2$ where $R_1 = R_2 = 1.25(119)^{1/3} = 6.1$ fm), the Coulomb barrier is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{R}$$

$$= (1.44 \text{ MeV} \cdot \text{fm})\frac{(46)^2}{12.2 \text{ fm}} = 250 \text{ MeV}$$

If we regard the zero of our energy scale to be the two fragments at rest separated by an infinite distance, then we can represent this system by Figure 13.1. Inside the region of nuclear potential, ^{238}U can exist as two ^{119}Pd nuclei because of the enormous number of final states accessible with a 214-MeV energy release. However, the Coulomb barrier prevents the fragments from separating, and the decay probability is small because the barrier cannot be penetrated.

This very crude calculation may indicate why fission is inhibited from readily occurring, but it should not be taken too seriously because the numbers we have used (250 MeV for the barrier height and 214 MeV for the energy release) are only estimates and can easily be modified by 10 to 20%. For example, the assumption that ^{238}U splits into two identical fragments may not be very realistic. If the two fragments have masses and atomic numbers in roughly a 2 : 1 ratio, such as $^{79}_{30}Zn$ and $^{159}_{62}Sm$, the Coulomb barrier height is reduced from 250 to 221 MeV. The release of a few neutrons will change the mass numbers of the final fragments and can produce more nearly stable and tightly bound fragments (nuclei such as ^{119}Pd, ^{79}Zn, and ^{159}Sm have a large neutron excess and are unlikely to be formed in fission). Also, the Coulomb barrier calculation based on a sharp edge at $R = R_1 + R_2$ is quite unlikely to be strictly correct.

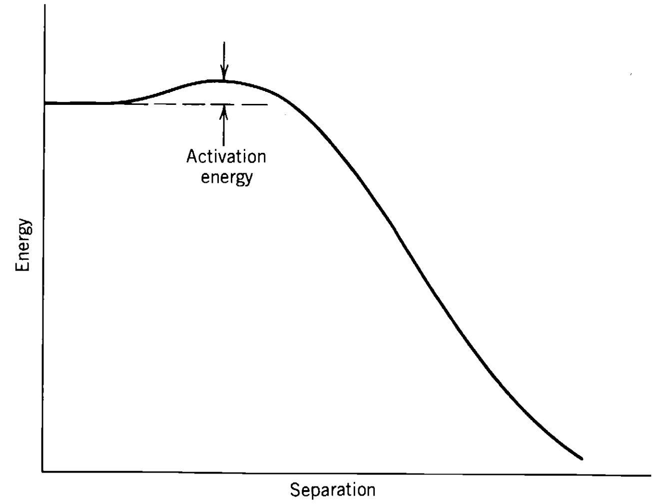

Figure 13.2 A smooth potential barrier opposing the spontaneous fission of ^{238}U. To surmount the fission barrier, we must supply an amount of energy equal to the activation energy.

What is certainly true, though, is that the height of the Coulomb barrier is *roughly* equal to the energy released in fission of heavy nuclei, and there are certain to be *some* nuclei for which the energy release puts the two fragments just below the Coulomb barrier, giving them a reasonably good chance to penetrate. These are the *spontaneously fissioning* nuclei, for which fission competes successfully with other radioactive decay processes. There may be others whose separated state would put them *above* the barrier and that would (if formed) instantly spontaneously fission. Such nuclei of course do not exist in nature; calculations suggest that the barrier against fission is zero at about $A = 300$. Still other nuclei may be far enough below the barrier that spontaneous fission is not observed, but absorption of a relatively small amount of energy, such as from a low-energy neutron or photon, forms an intermediate state (perhaps a compound nuclear state) that is at or above the barrier so that *induced fission* occurs readily, that is, it competes successfully with other modes of decay of the compound nucleus. If the intermediate state is below the barrier, fission is inhibited and other decay modes, including re-emission of the absorbed particle, may dominate. Subthreshold fission may have important implications for nuclear structure for there are often resonances that can enhance the fission probability, as we discuss in Section 13.4. The ability of a nucleus to undergo induced fission will depend critically on the energy of the intermediate system; for some nuclei, absorption of thermal neutrons may be sufficient to push them over the barrier, while for others, fast (MeV) neutrons may be required. Figure 13.2 shows a more realistic representation of the fission barrier for heavy nuclei.

A more detailed calculation of the energy needed to induce fission is shown in Figure 13.3, which gives essentially the height of the fission barrier above the ground state (usually called the *activation energy*). This calculation is based on the liquid-drop model, which treats only average nuclear properties; the inclusion of more sophisticated effects based on the shell model modifies the calculation